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TECHNICAL NOTE

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THERMODYNAMIC AND TRANSPORT PROPERTY CORRELATION FORMULAS FOR EQUILIBRIUM AIR FROM 1,000° K TO 15,000° K

By John R. Viegas and John T. Howe

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SUMMARY

The thermodynamic properties, density and temperature, as well as transport property parameters involving viscosity, Prandtl number (including diffusion effects), and gaseous radiation absorption coefficients have been correlated as a function of enthalpy at four pressure levels (10^{-1}, 10^{0}, 10^{1}, and 10^{2} atmospheres). The correlation formulas are written in a generalized form for which coefficients for a particular property and pressure level are tabulated. The correlation formulas are useful in digital computer programs for nonadiabatic viscous flow problems.

INTRODUCTION

Thermodynamic and transport properties of high temperature air as well as their derivatives with respect to enthalpy at constant pressure are often needed for the computation of flow fields on bodies in high-speed flight. These properties are available in tabular form (refs. 1 and 2). However, this form is often not very convenient for use in digital computers. To facilitate machine computations, it is sometimes faster and easier if the properties are represented by analytical expressions which can also be readily differentiated. Cohen (ref. 3) correlates density and some transport properties independently of pressure for flight speeds up to 29,000 ft/sec. Correlations are now required for high speeds up to 50,000 ft/sec to facilitate studies of high-speed entry into the earth's atmosphere.

At these higher speeds, in excess of approximately 30,000 feet per second, the transport properties of air are significantly affected by ionization. Furthermore, at these speeds, gaseous radiation effects can also be important, depending on body size and altitude (ref. 4). Thus, for entry into the earth's atmosphere on return from the moon, the planets, or far out in the solar system, for which entry speeds will be between 35,000 and 50,000 feet per second, both ionization and radiation effects may be important and should be considered. For these reasons, thermodynamic and transport properties of equilibrium air at temperatures up to 15,000° K (stagnation temperature for flight at 50,000 ft/sec at approximately 190,000 ft altitude) as presented in references 1 and 2, are correlated as functions of enthalpy at the four pressure levels, 10^{-1}, 10^{0}, 10^{1}, 10^{2} atmospheres in the present work. The Planck mean mass absorption coefficient for gaseous radiation, as presented by both references 5 and 6, is also correlated as functions of enthalpy at the same pressure levels.

SYMBOLS

a,b,c,d,e	constant and coefficients in equation (1)
h	enthalpy, ft^2/sec^2
k	Planck mean mass absorption coefficient, $ft^2/slug$
p	pressure, atm
Pr	Prandtl number
T	temperature, °K
x	independent variable in equation (1), $\frac{h}{h_r}$
y	dependent variable in equation (1) (appropriate property)
ρ	mass density, $slug/ft^3$
μ	viscosity coefficient, $lb\ sec/ft^2$

Subscripts

a	results obtained from reference 5
l	results obtained from reference 6
1,2, . . . n	e coefficients in equation (1)
r	reference conditions in table II.

CORRELATION FORMULAS

Many attempts were made to fit smooth curves through the desired property values obtained from references 1, 2, 5, and 6. Polynomials of all degrees up to 8 with coefficients determined by the method of least squares and the method of Tchebycheff were tried. Fourier series and generalized conics were also tried. In some cases it was found necessary to join as many as four sections of curves smoothly in series to obtain adequate correlations. An attempt was made to minimize possible discontinuities at the joints in segmented curves either by overlapping the sections and choosing a suitable point in this overlapped region to be the limit of the various curves (this was done for conics), or by matching the slope and property value of two adjoining curves at the same value of enthalpy (this was done for polynomials). The curves presented in this paper are the best results, from the methods attempted, for obtaining accurate and smoothly varying property values as functions of enthalpy.

Although various methods of correlation are used, it is convenient to express the correlation of all properties at a specific pressure level by the general formula

$$a + by + cxy + dy^2 + e_1x + e_2x^2 + e_3x^3 \ . \ . \ . \ e_nx^n = 0 \qquad (1)$$

where the independent variable x is the enthalpy ratio h/h_r and the dependent variable y is the appropriate gas property. It is seen that if the coefficients $e_3 \ . \ . \ . \ e_n$ are zero, the equation is that of a general conic with inclined axis, and if the coefficients c and d are zero, the equation is that of a polynomial of degree n.

To facilitate the use of equation (1), the coefficients for the various properties at the pressure levels considered are presented in table I. This table shows the type of correlating function used (general conic or polynomial of degree n) for each property, the upper and lower enthalpy limits for the validity of each section of the correlation curve, and, for those properties fit by a general conic, the sign of the appropriate root is also given.

The over-all limits of validity of these correlation formulas for each property correspond to a temperature range of approximately 1,000° to 15,000° K. For convenience, each property is referenced to a standard condition. The pressure is referenced to sea-level conditions. The reference conditions of all other properties correspond to their values at satellite enthalpy ($h_r = 3.125\times10^8$ ft^2/sec^2 or 12,474 Btu/lb) at each pressure level. The reference conditions are listed in table II.

DISCUSSION OF RESULTS

The thermodynamic and transport properties as obtained from the correlation formulas presented in this report and the properties they represent from references 1, 2, 5, and 6 are compared in figures 1 through 5. In general, the agreement is good. The analytical expressions should provide property values with sufficient accuracy for most machine calculations. In the remainder of this discussion, consideration is given to certain features of the correlations.

In figure 3 where the ratio of the density-viscosity product to the Prandtl number is correlated, no attempt was made to fit the minor variation in the property values which occurred at low enthalpies (near $h/h_r = 0.3$). It was felt that the effect of this variation would be negligible in comparison to the effects of the over-all variation. The peak in each curve corresponds to the onset of ionization. The correlation is seen to be very good in the ionization regime.

To study the effects of energy transport by gaseous radiation, the Planck mean mass absorption coefficient is useful. It has been calculated from theory in reference 6 and has been obtained by a combination of theory and experiment in reference 5. The two references are in reasonable, but not close, agreement. The results of both are correlated in figure 5.

First the absorption coefficient of reference 6 as correlated for all pressure levels is shown in figure 5(a). The fit is fair except for the point at $h/h_r \approx 3.9$ for 1 atmosphere pressure. The high point at each pressure level corresponds to 15,000° K and was obtained by a graphical and logarithmic interpolation of results in reference 6 at 12,000° and 18,000° K.

The correlation of the absorption coefficient at individual pressure levels is shown in figures 5(b) through 5(e) corresponding to the results of reference 6 and figure 5(f) through 5(i) corresponding to the results of reference 5. Except for figure 5(f), the correlation is satisfactory. No attempt was made to fit figure 5(f) because of the lack of a point defining the middle range of the properties.

Finally, it is instructive to go back and examine the thermal conductivity used in the Prandtl number of figure 3. This is especially pertinent because the current lack of agreement between the stagnation point convective heat-transfer rates in references 7 and 8 may be attributed to the thermal conductivity of ionized air.

In the present paper, the thermal conductivity used in the Prandtl number includes the effects of energy transfer by both molecular collisions and diffusion of molecular species (ref. 2). The thermal conductivity calculated for air by Hansen (ref. 2) agrees quite well with experimental results at temperatures up to 5,000° K as reported by Peng and Ahtye (ref. 9). At higher temperatures, Hansen's results can be compared with the conductivity deduced experimentally for nitrogen by Maecker (ref. 10). In this comparison, shown in figure 6, agreement is fairly good and the relative magnitudes of conductivities of the two gases are as expected (see refs. 9 and 11). Results of King (ref. 12) for the thermal conductivity of pure nitrogen agree very well with those of Maecker.

CONCLUDING REMARKS

Thermodynamic properties and transport property parameters have been correlated as a function of enthalpy at four pressure levels. In general, the correlation formulas represent the properties quite accurately. Although the formulas are lengthy, they can be evaluated very rapidly by digital computers. For example, a property represented by an eighth-degree polynomial can be evaluated at 1000 points in approximately 0.7 second on an IBM 7090 data processing machine. This is one to two orders of magnitude faster than having this machine look up the same number of points in a table. Thus, the formulas are expected to be useful for flow-field computation on digital computers.

Ames Research Center
National Aeronautics and Space Administration
Moffett Field, Calif., June 21, 1962

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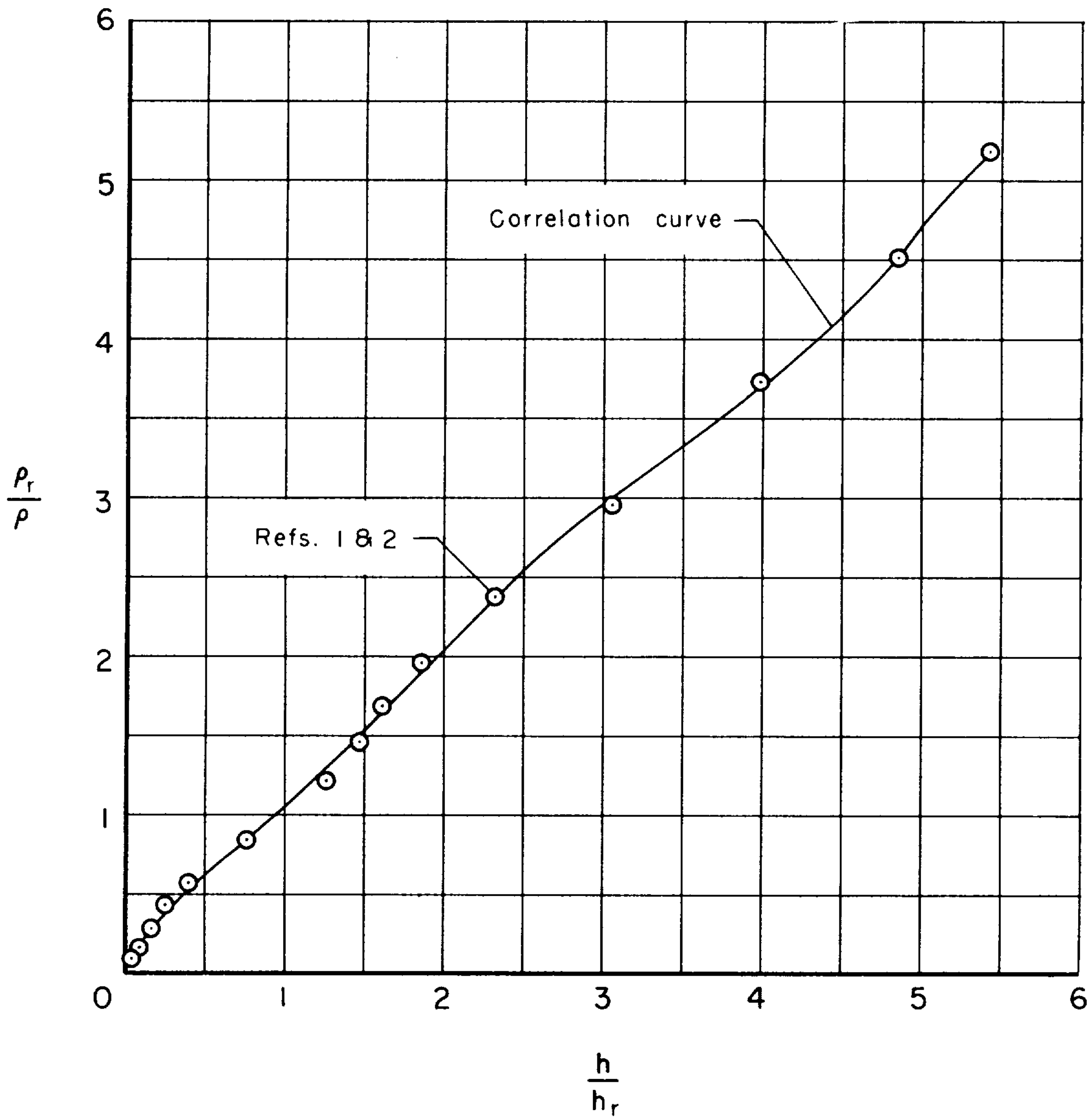
TABLE I.- COEFFICIENTS

Property, y	Pressure level, atm	Type of curve	x limits for curve Lower	x limits for curve Upper	a	b	c	d	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	Root of quadratic
$\frac{\rho_r}{\rho}$	10^{-1}	(a)	0.03608	5.4273	0.06592768	-1	0	0	1.505904	-1.165208	0.9343309	-0.3375398	0.05513693	-0.003330603	0	0	
	10^{0}	(a)		3.9643	.007398195	-1			2.109765	-2.739214	2.364413	-.5556323	.1007640	.009547003	-.002160125	0	
	10^{1}	(b)		2.6736	.01948670	-1			1.329233	-2.967630	7.119123	-11.11281	9.714190	-4.594607	1.105638	-.1064150	
	10^{2}	(a)		2.1589	.01752455	-1			1.533830	-.3134607	.5619979	-.3583186	.1964153	-.04129094	0	0	
$\frac{\rho\mu}{\rho_r\mu_r}$	10^{-1}	(c)		5.4273	7.999610	-5.624454	-2.135941	1	-1.005803	-.04987680	0	0	0	0			-
	10^{0}			3.9643	2.957720	-4.336691	12.54509	1	-15.59082	3.058515							+
	10^{1}			2.6736	-13.99580	-1.443177	89.74126	1	-89.58862	15.39430							+
	10^{2}			2.1589	.1344780	-4.118392	28.04161	1	-30.05294	4.841396							+
$\frac{\rho\mu Pr_r}{\rho_r\mu_r Pr}$	10^{-1}			1.4	15.16115	-7.702873	2.738835	1	-19.90980	7.949521							-
			1.4	1.6	-.9376520	1	-.5666152	0	.8101086	-.2057187							
			1.6	1.835	25.02171	-9.583479	3.787318	1	-19.55076	3.657150							+
			1.835	5.4273	-5.528132	1	1.029237	0	.0006060	.06293130							
	10^{0}		.03608	.45	-4.674994	1	12.72930	0	-19.98398	9.350414							
			.45	1.6	108.2544	-45.80435	23.90831	1	-130.1215	42.30918							-
			1.55	2.35	-33.45746	8.832652	-7.821374	1	30.56290	-4.302969							-
			2.35	3.9643	-3.994343	1	.3394629	0	.5779776	-.03073330							
	10^{1}		.03608	1.0	14.04183	-7.002485	-24.27600	1	41.25561	-25.31620							-
			.80	1.64	-172.3071	72.84828	-39.60441	1	203.6301	-64.21053							+
			1.6	2.6736	-17.96245	6.135884	-5.637358	1	15.14080	-1.694240							-
	10^{2}		.03608	.68	8.065993	-6.511489	50.27557	1	-109.5963	59.75427							+
			.68	1.5	-2.468844	1	.334055	0	1.417372	-.3146546							
			1.5	1.81	-24.14852	16.07877	-9.454224	1	22.63899	-5.168075							+
			1.78	2.16	-17.93436	9.928399	-7.072455	1	11.35373	-.1983110							-
$\frac{T}{T_r}$	10^{-1}	(a)	.03608	.08	-.07161016	-1	0	0	7.595930	-35.45260							
			.08	5.4273	.1364769	-1			2.411026	-3.378803	2.901134	-1.302484	.3106245	-.03751842	.0018102816		
	10^{0}		.03608	2.54	.01726946	-1			3.500558	-6.042997	5.382014	-2.327105	.4793450	-.03780220	0		
		(c)	2.54	3.9643	1.243210	-1			.2121646	0	0	0	0	0			
	10^{1}	(a)	.03608	2.6736	.05531397	-1			2.455306	-2.249730	.2606849	.8472206	-.4429157	.06444144			
	10^{2}	(a)		2.1589	.03401435	-1			2.412343	-2.467107	1.219690	-.2029401	0	0			
$\left(\frac{k}{k_r}\right)_l$	10^{-1}	(c)		5.4273	0	68.85393	-33.18305	1	-37.97243	6.520246	0	0					+
	10^{0}			3.9643		25.01277	-18.74220	1	-15.02563	7.682671							+
	10^{1}			2.6736		133.9065	-51.91449	1	-27.90829	-49.49101							+
	10^{2}			2.1589		23.19744	-15.39758	1	-12.22303	7.645397							+
	All levels			5.4273		75.18458	-33.43958	1	-24.53530	-13.84419							+
$\left(\frac{k}{k_r}\right)_a$	10^{-1}			5.4273		0	0	0	0	0							
	10^{0}			3.9643		49.33528	-35.56380	1	-15.28951	8.139395							+
	10^{1}			2.6736		80.99429	-39.79852	1	1.248875	-39.13508							+
	10^{2}			2.1589		1634.040	-689.9332	1	-305.7013	-234.0838							+

[a]Polynomial (least squares)
[b]Polynomial (Tchebycheff)
[c]Conic

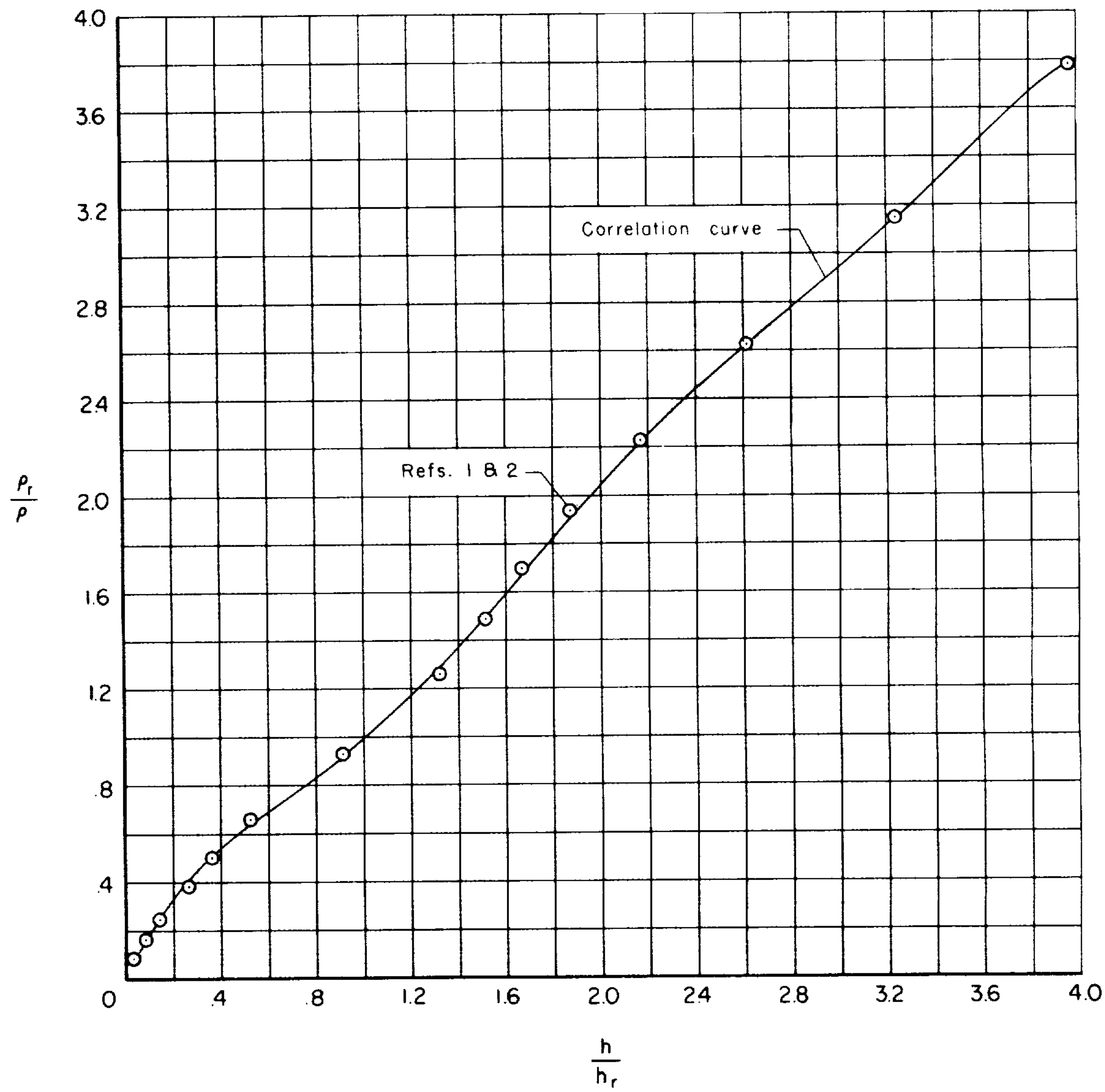
TABLE II.- REFERENCE CONDITIONS

Pressure level, atm	h_r, ft^2/sec^2	ρ_r, slug/ft^3	$\rho_r \mu_r$, lb^2 sec^3/ft^6	$\frac{Pr_r}{\rho_r \mu_r}$, ft^6/lb^2 sec^3	T_r, $^\circ$K	$(k_r)_l$, ft^2/slug	$(k_r)_a$, ft^2/slug
10^{-1}	3.125×10^8	0.6271×10^{-5}	1.8254×10^{-11}	0.4777×10^{11}	6400	1.66×10^1	2.42×10^1
10^0	3.125×10^8	$.5700 \times 10^{-4}$	1.8065×10^{-10}	$.4694 \times 10^{10}$	7200	6.68×10^1	9.68×10^1
10^1	3.125×10^8	$.5185 \times 10^{-3}$	1.7754×10^{-9}	$.4613 \times 10^9$	8150	2.11×10^2	3.87×10^2
10^2	3.125×10^8	$.4697 \times 10^{-2}$	1.7556×10^{-8}	$.4460 \times 10^8$	9350	8.62×10^2	1.82×10^3



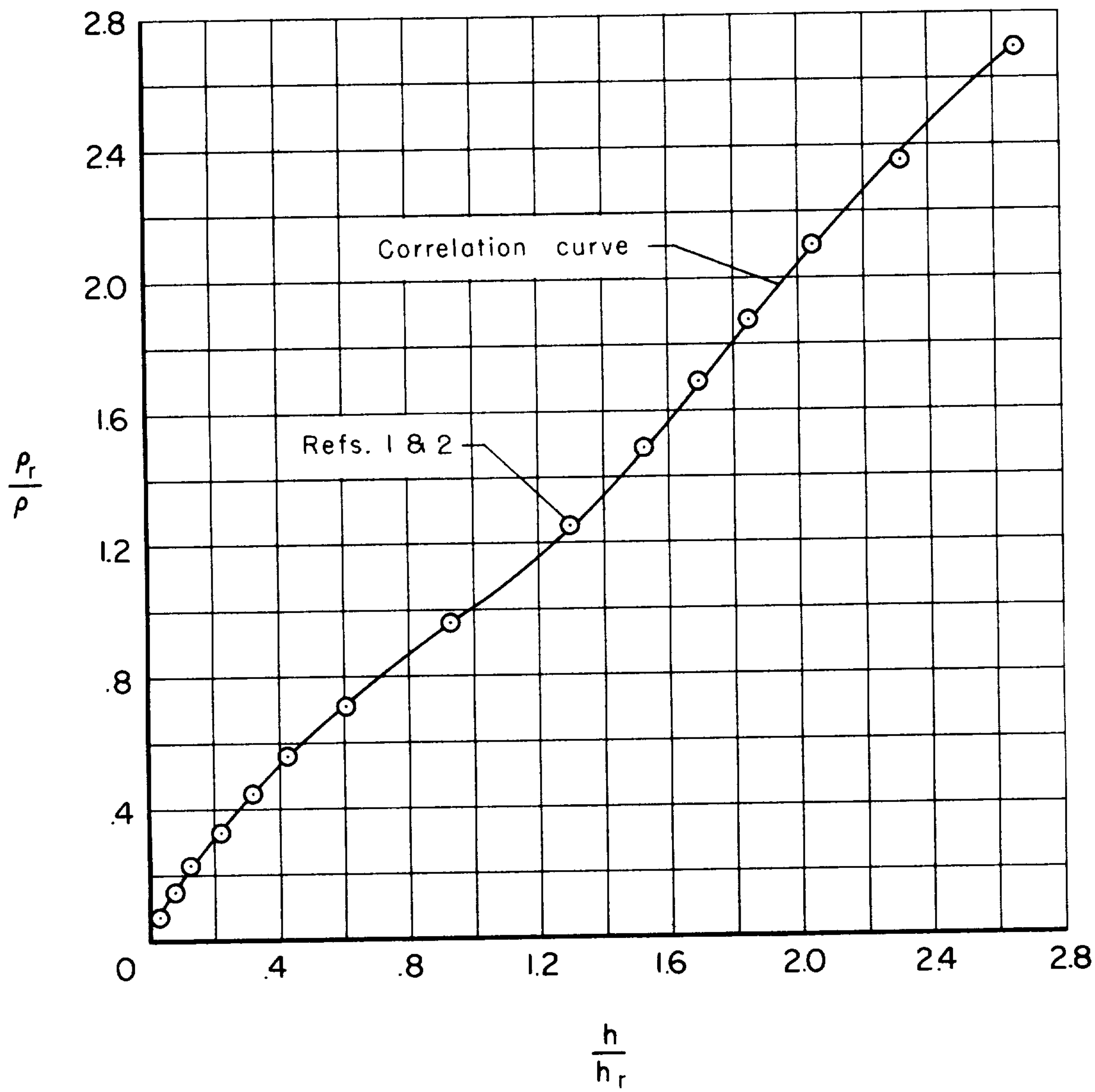
(a) $p = 10^{-1}$ atm

Figure 1.- Density correlation.



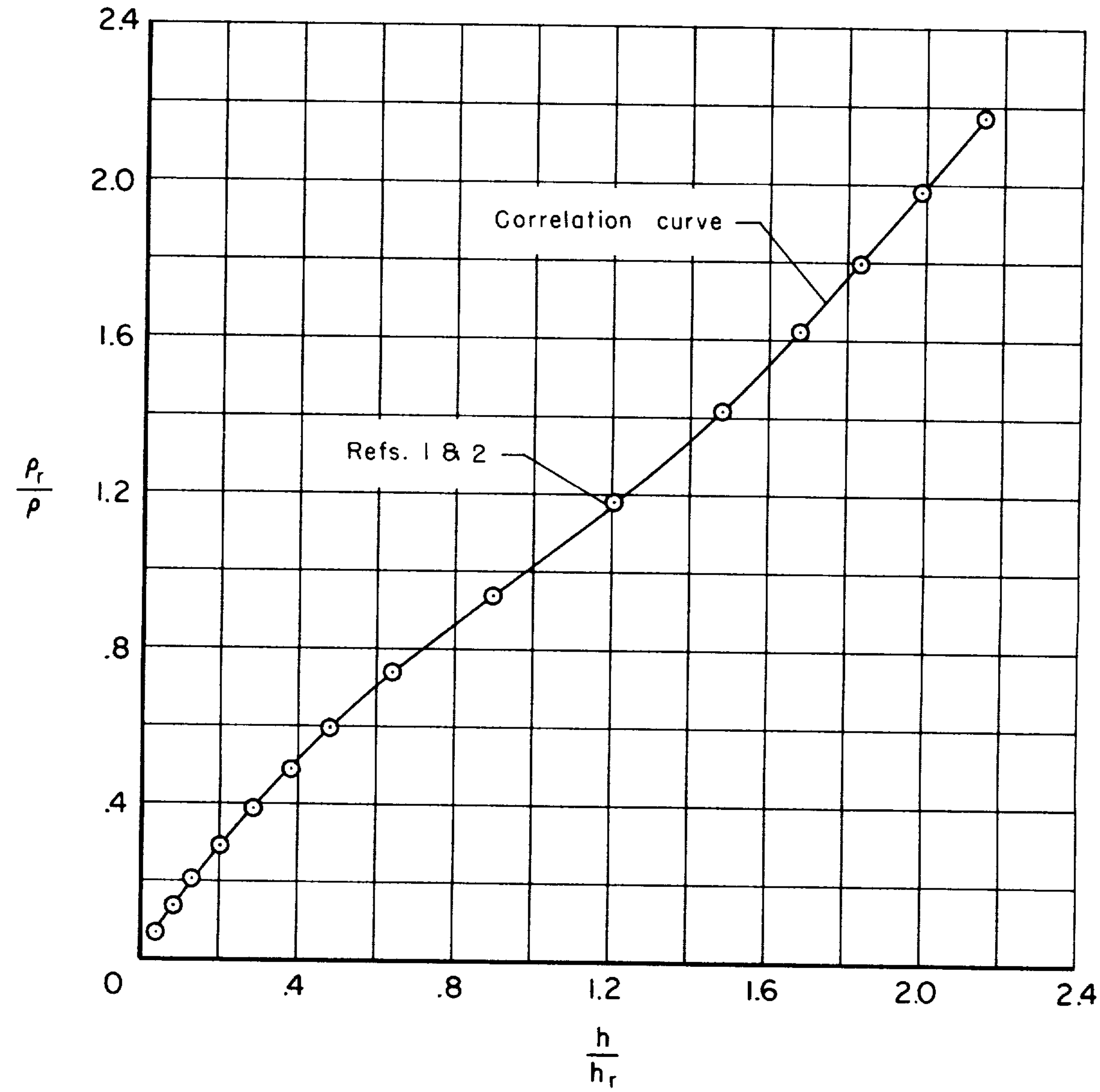
(b) p = 1 atm

Figure 1.- Continued.



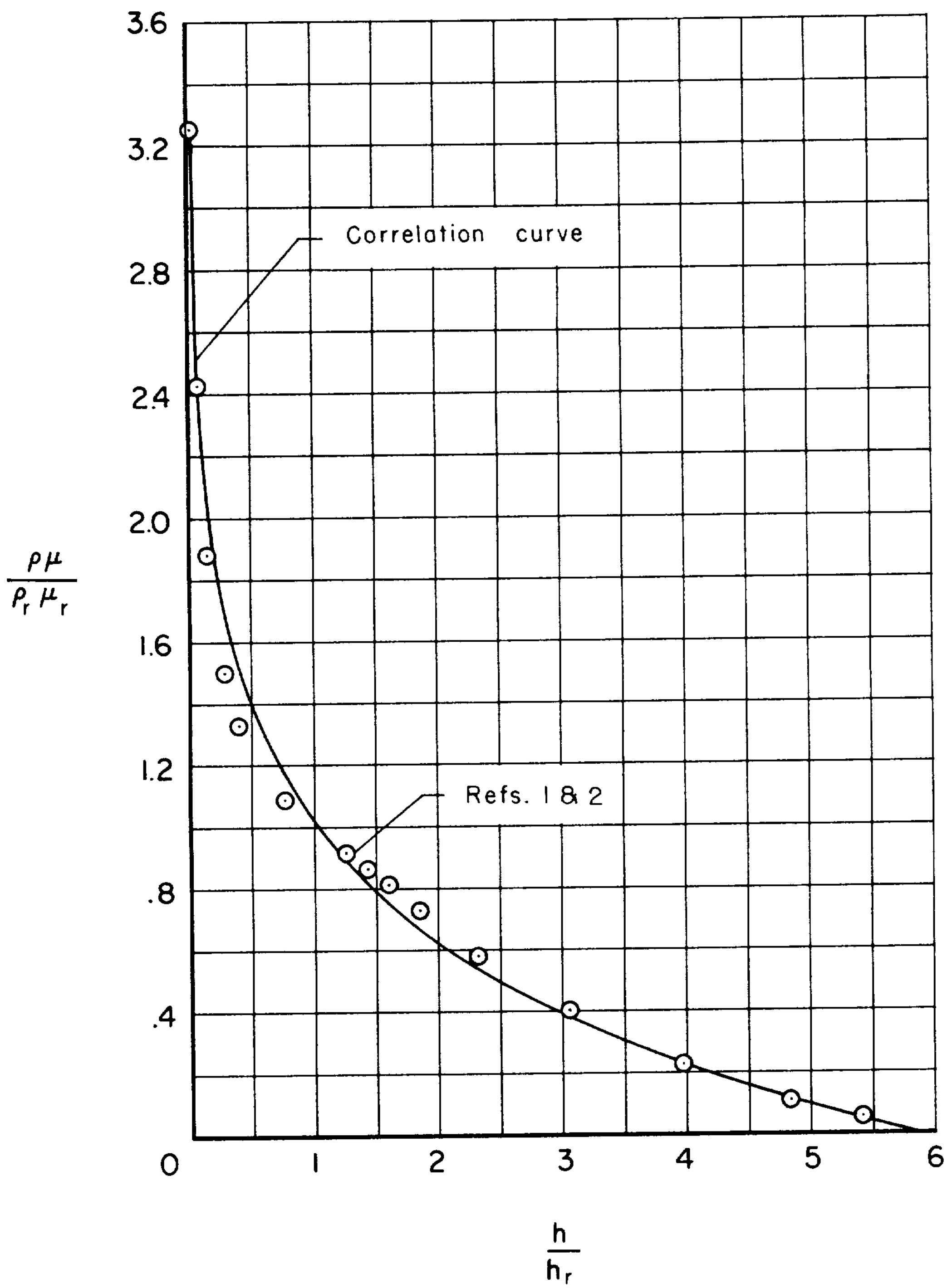
(c) p = 10 atm

Figure 1.- Continued.



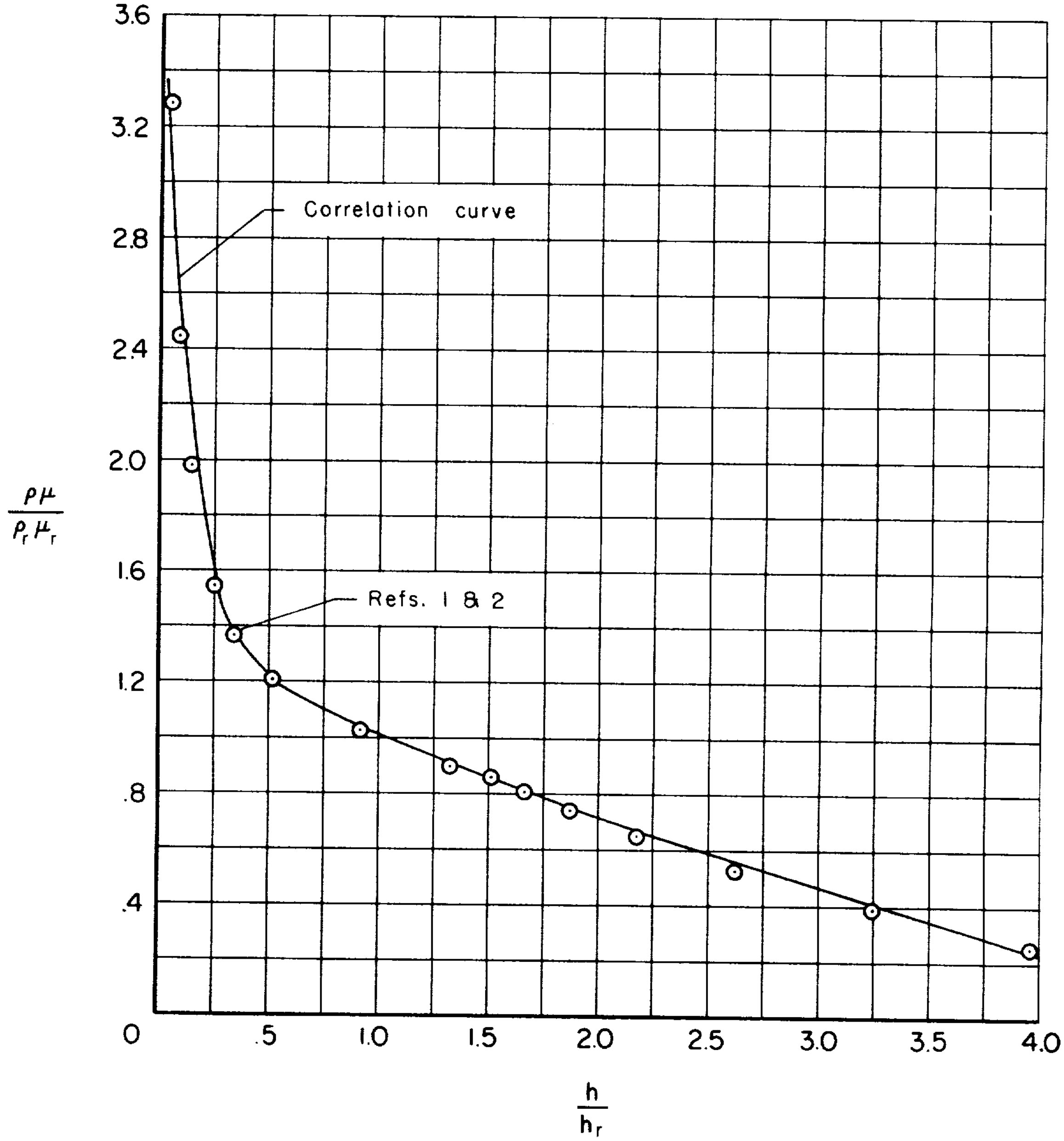
(d) p = 100 atm

Figure 1.- Concluded.



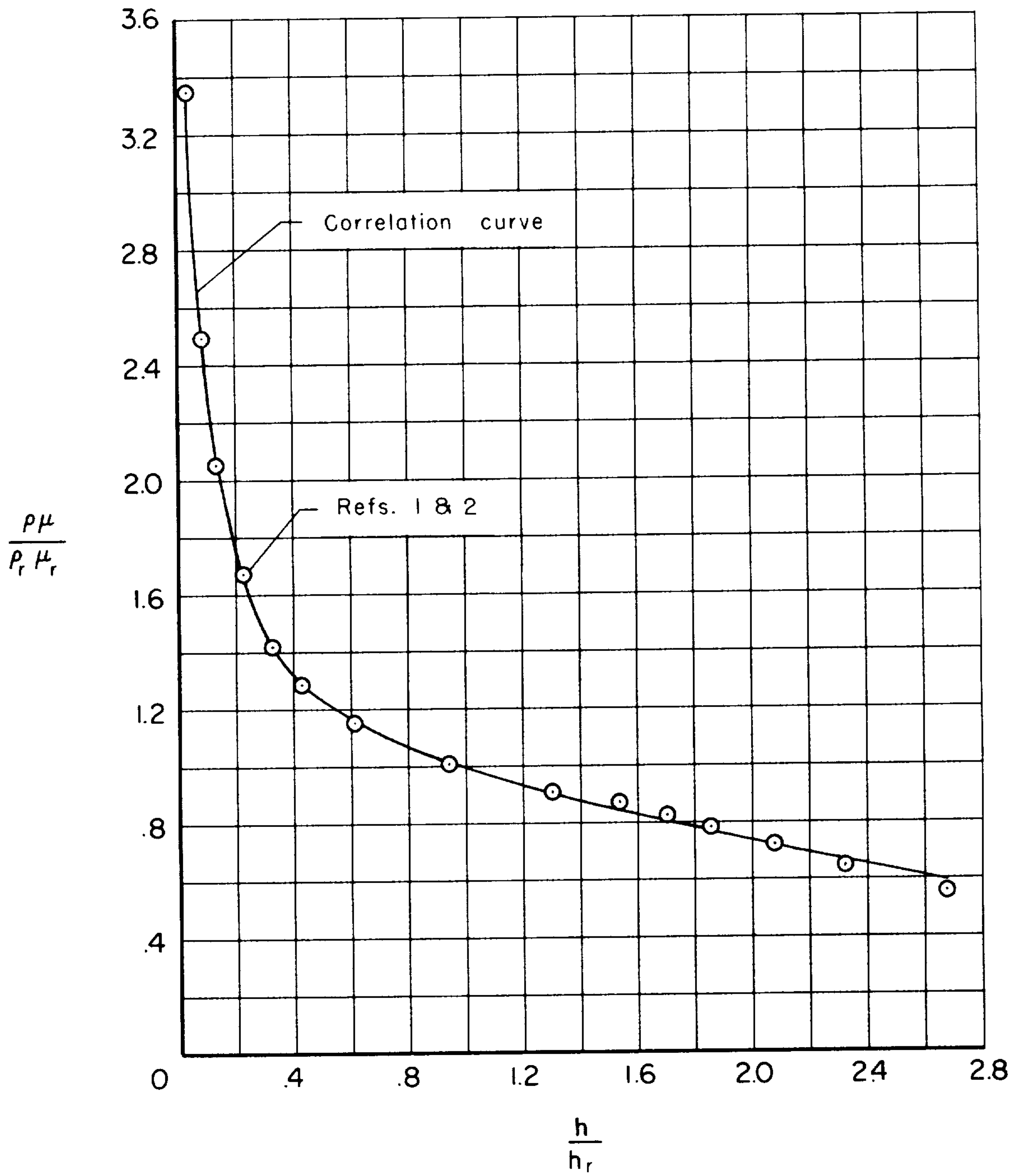
(a) $p = 10^{-1}$ atm

Figure 2.- Density-viscosity parameter correlation.



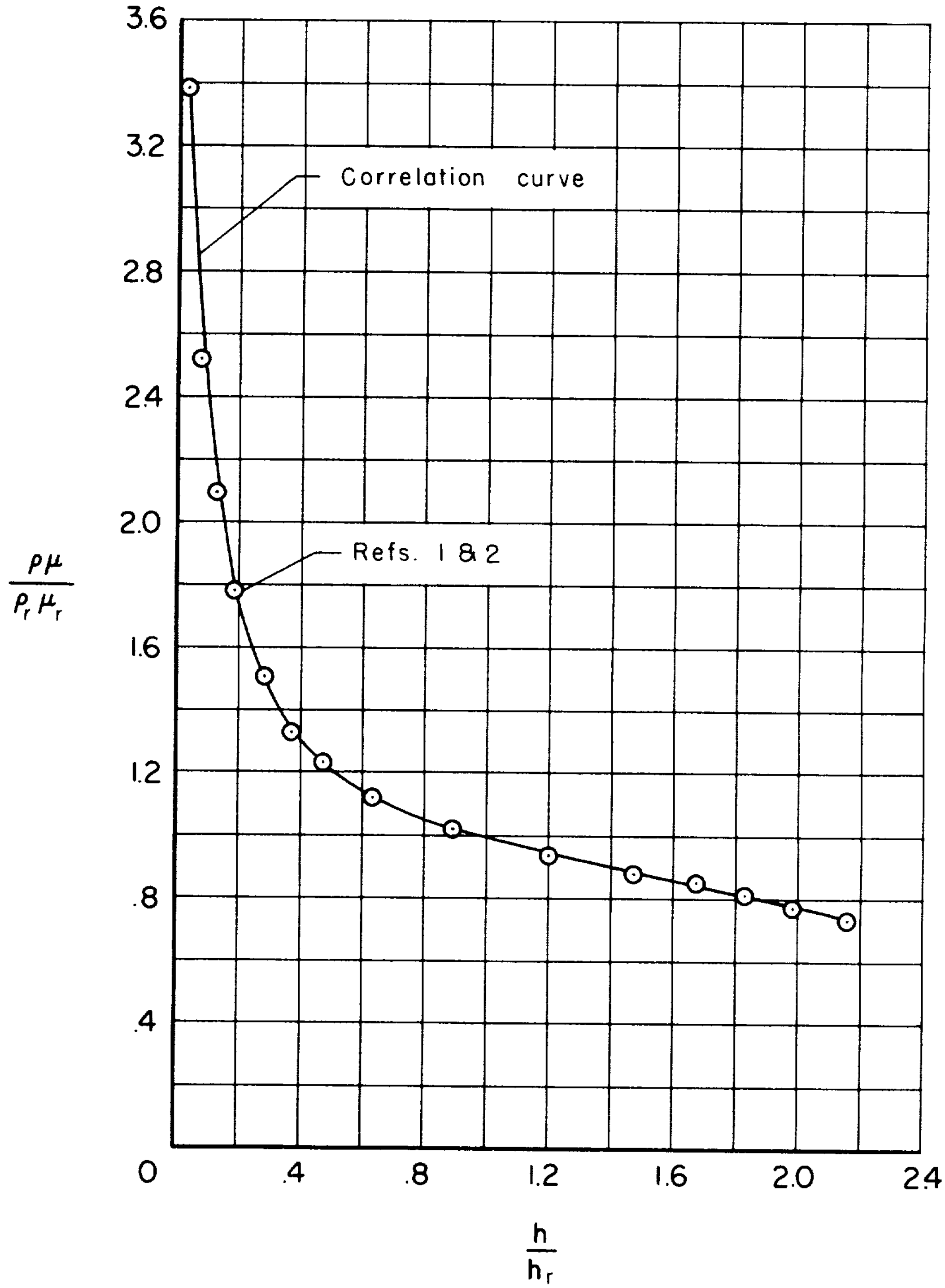
(b) p = 1 atm

Figure 2.- Continued.



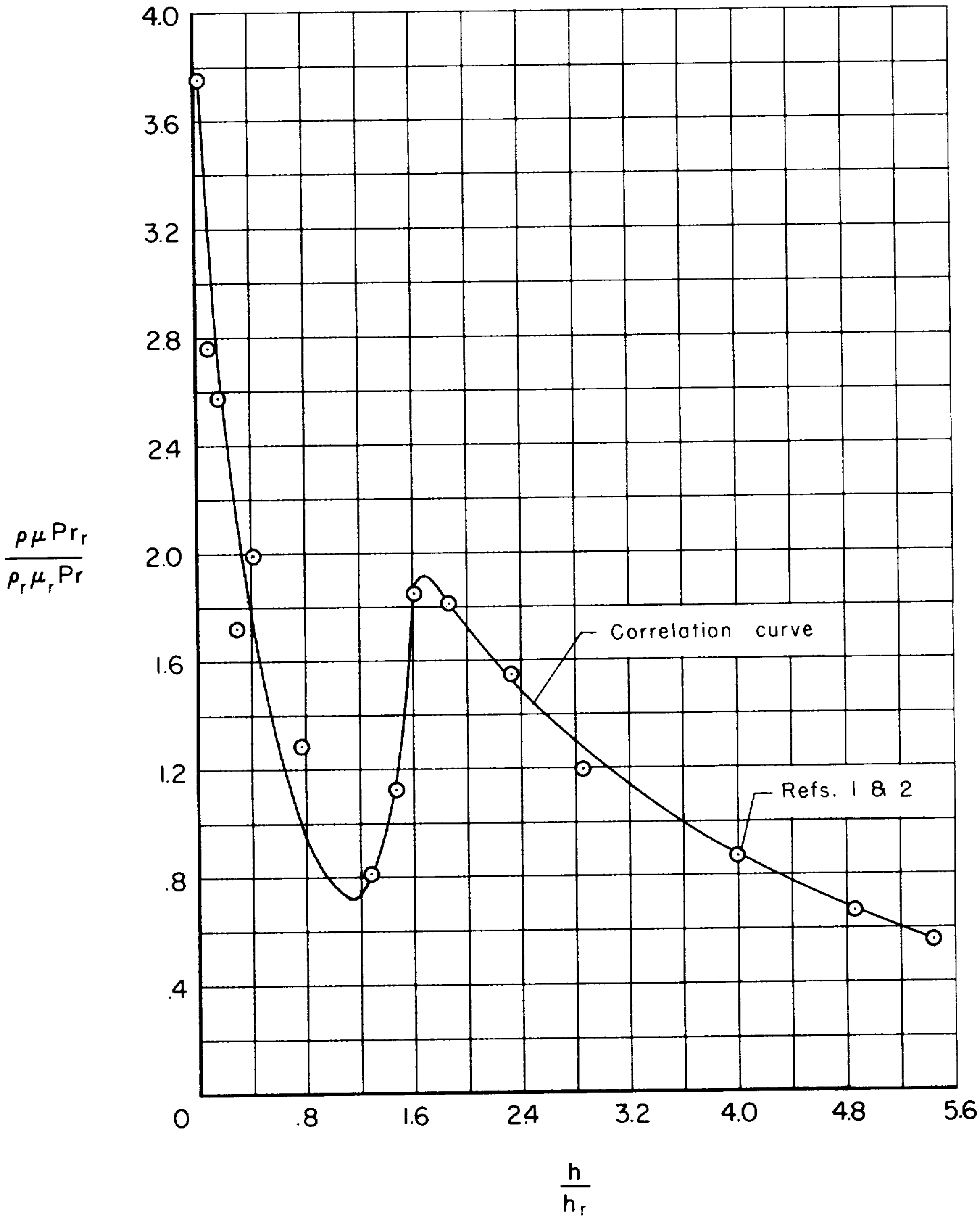
(c) p = 10 atm

Figure 2.- Continued.



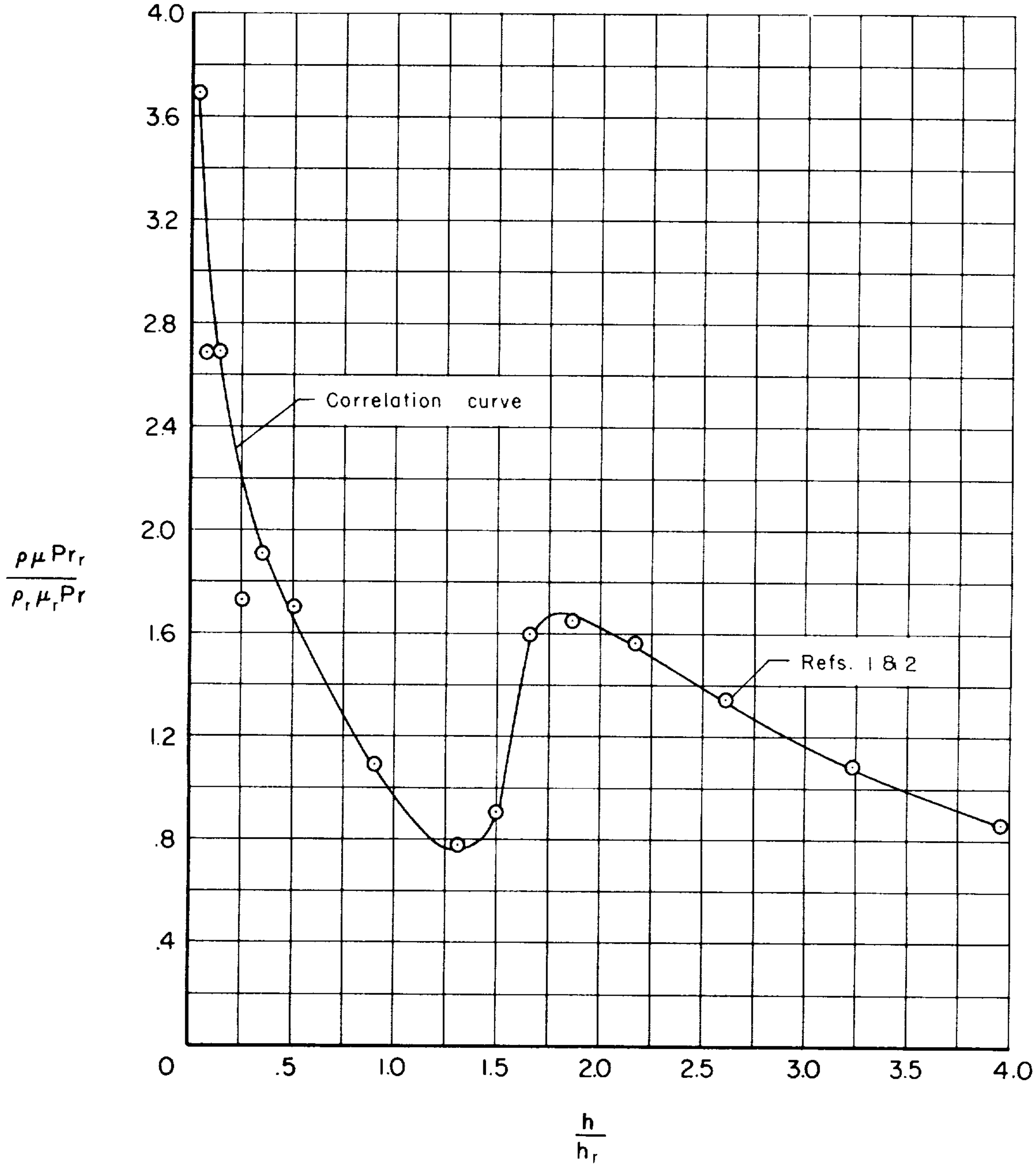
(d) p = 100 atm

Figure 2.- Concluded.



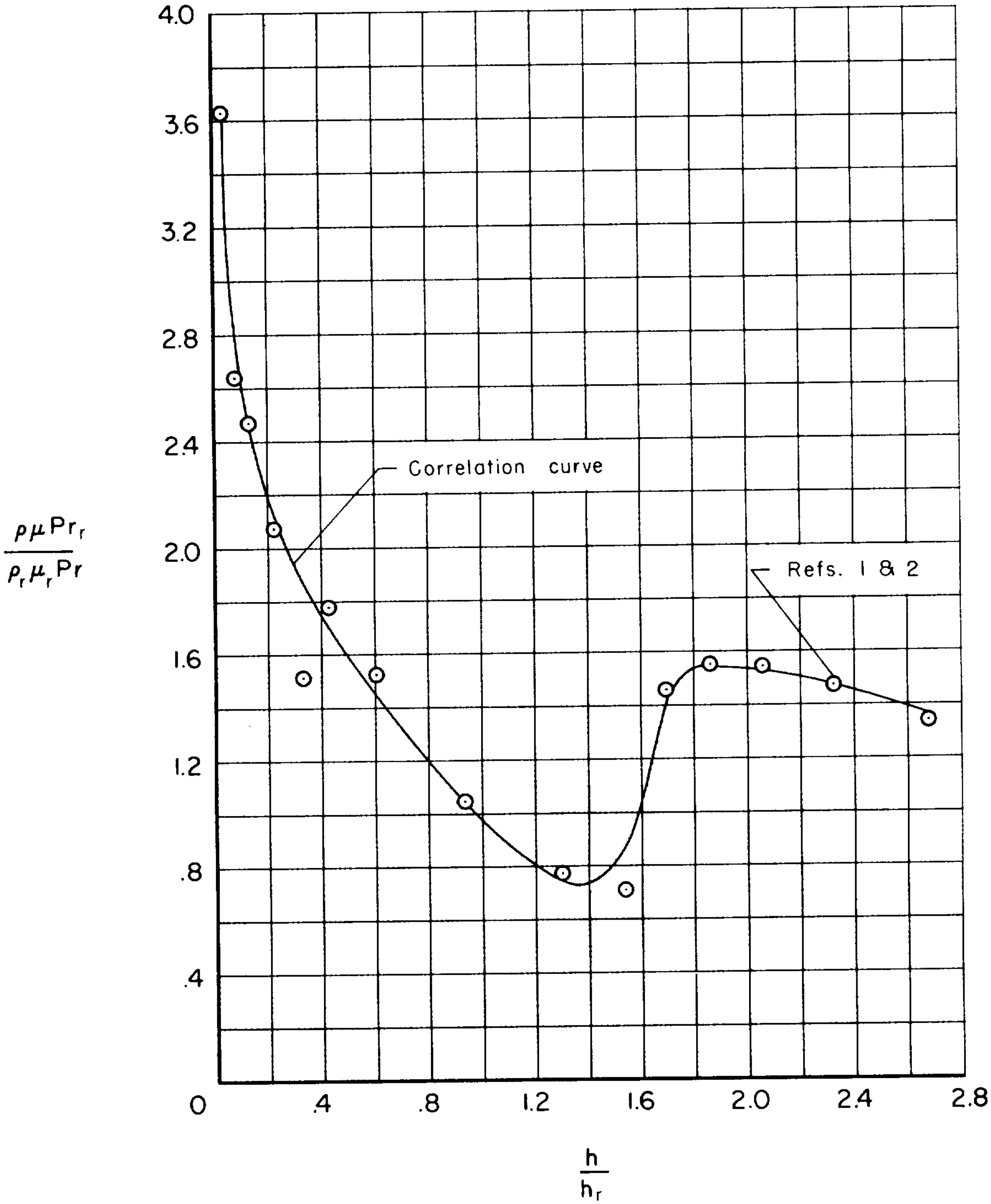
(a) $p = 10^{-1}$ atm

Figure 3.- Density-viscosity Prandtl number parameter correlation.



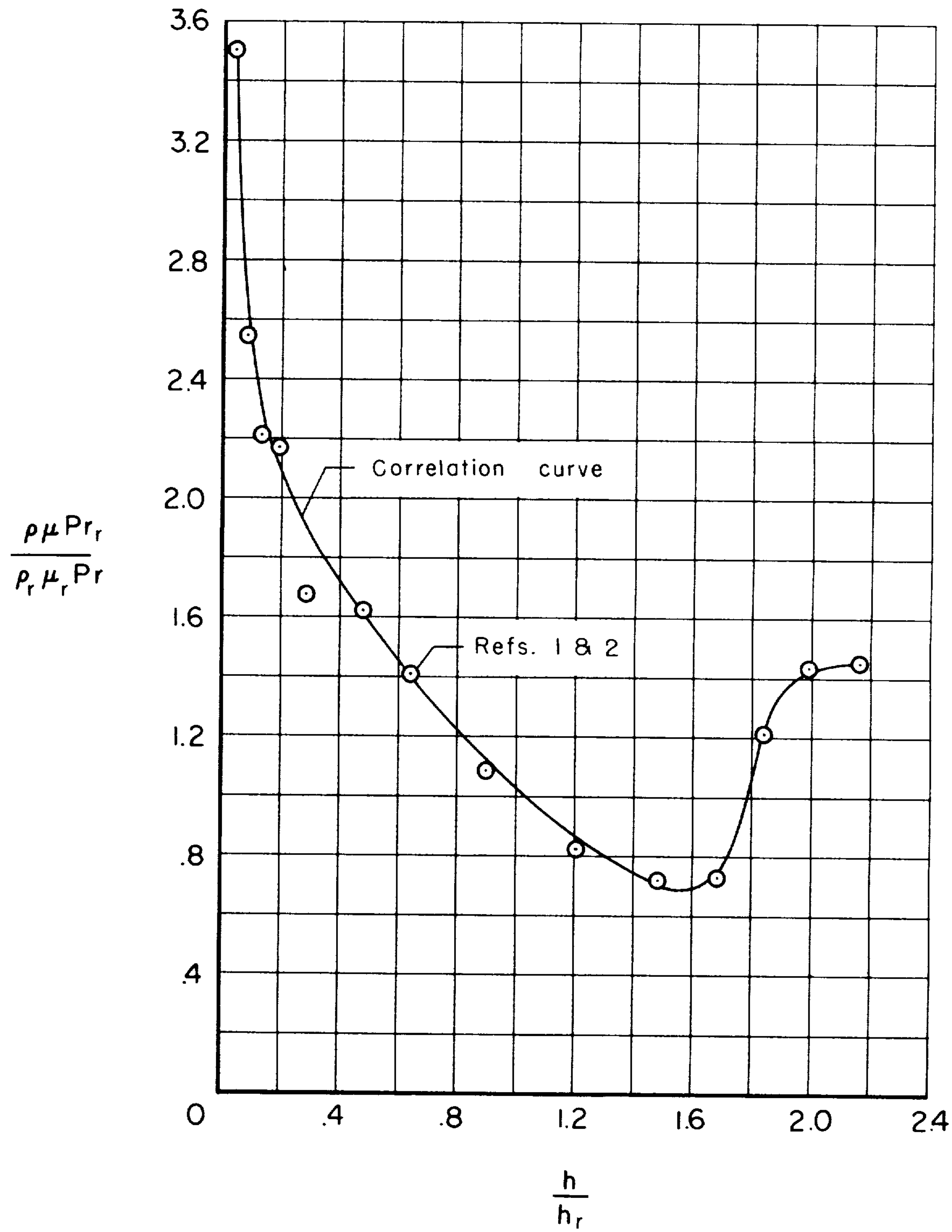
(b) p = 1 atm

Figure 3.- Continued.



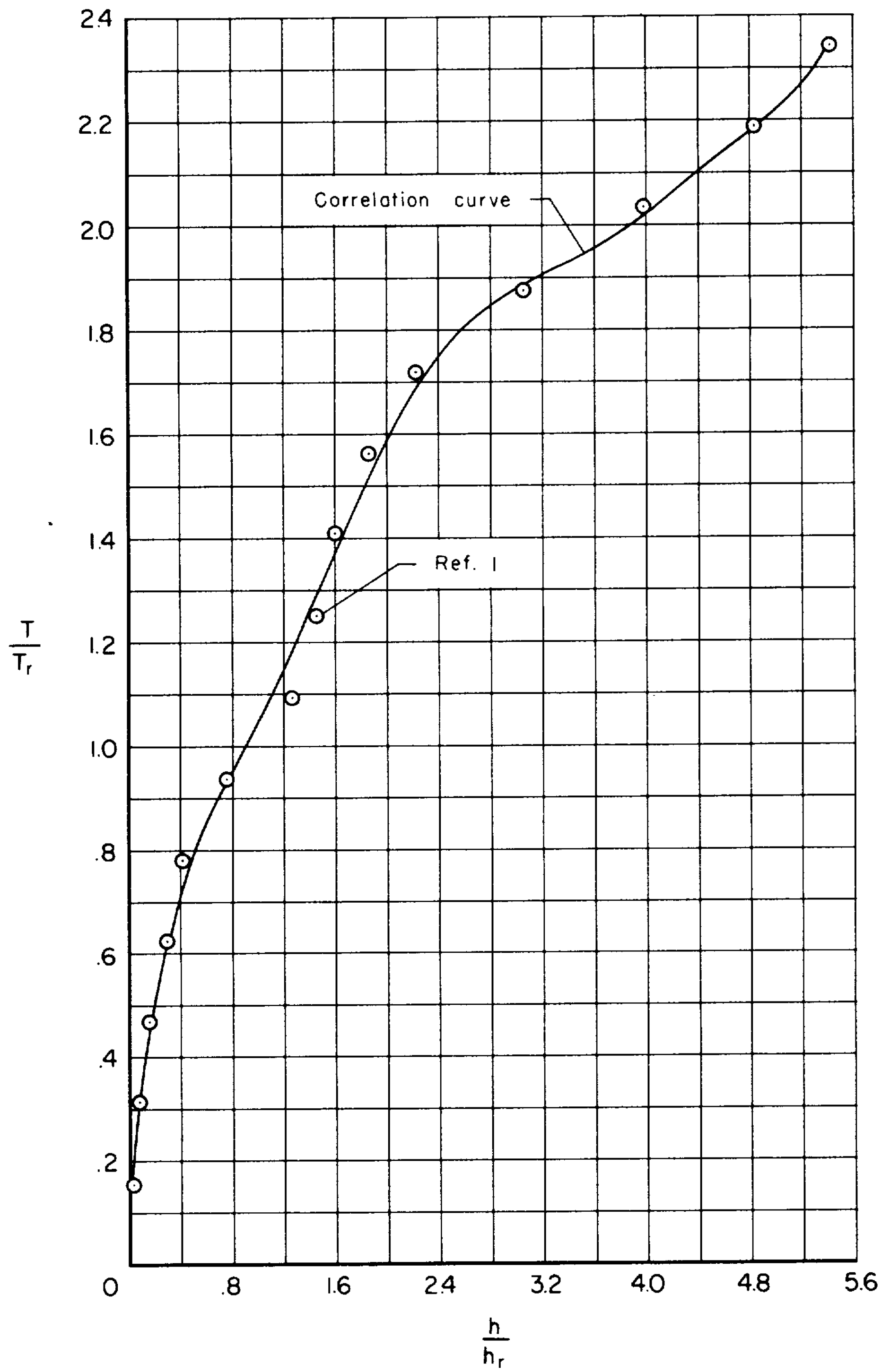
(c) p = 10 atm

Figure 3.- Continued.



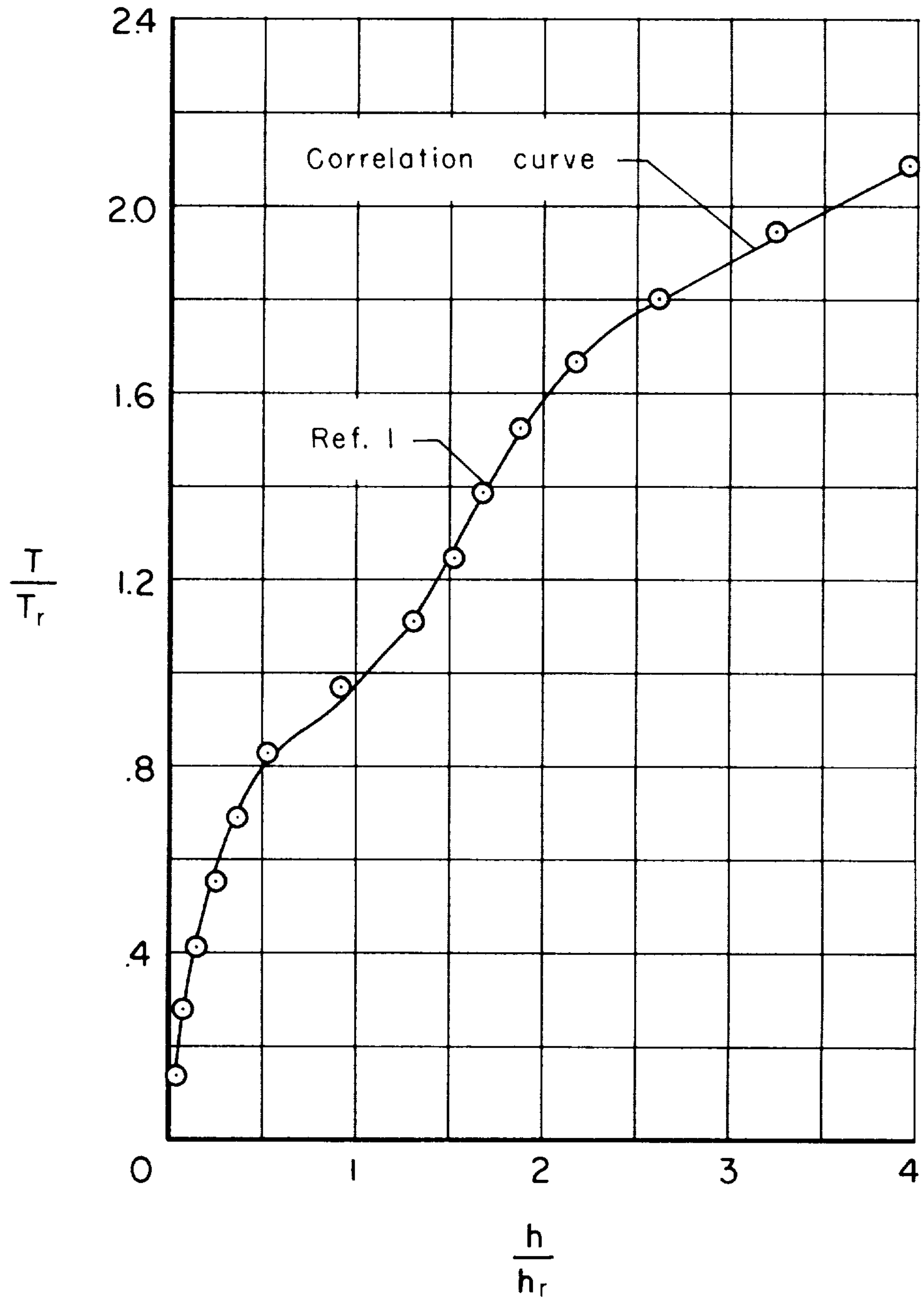
(d) p = 100 atm

Figure 3.- Concluded.



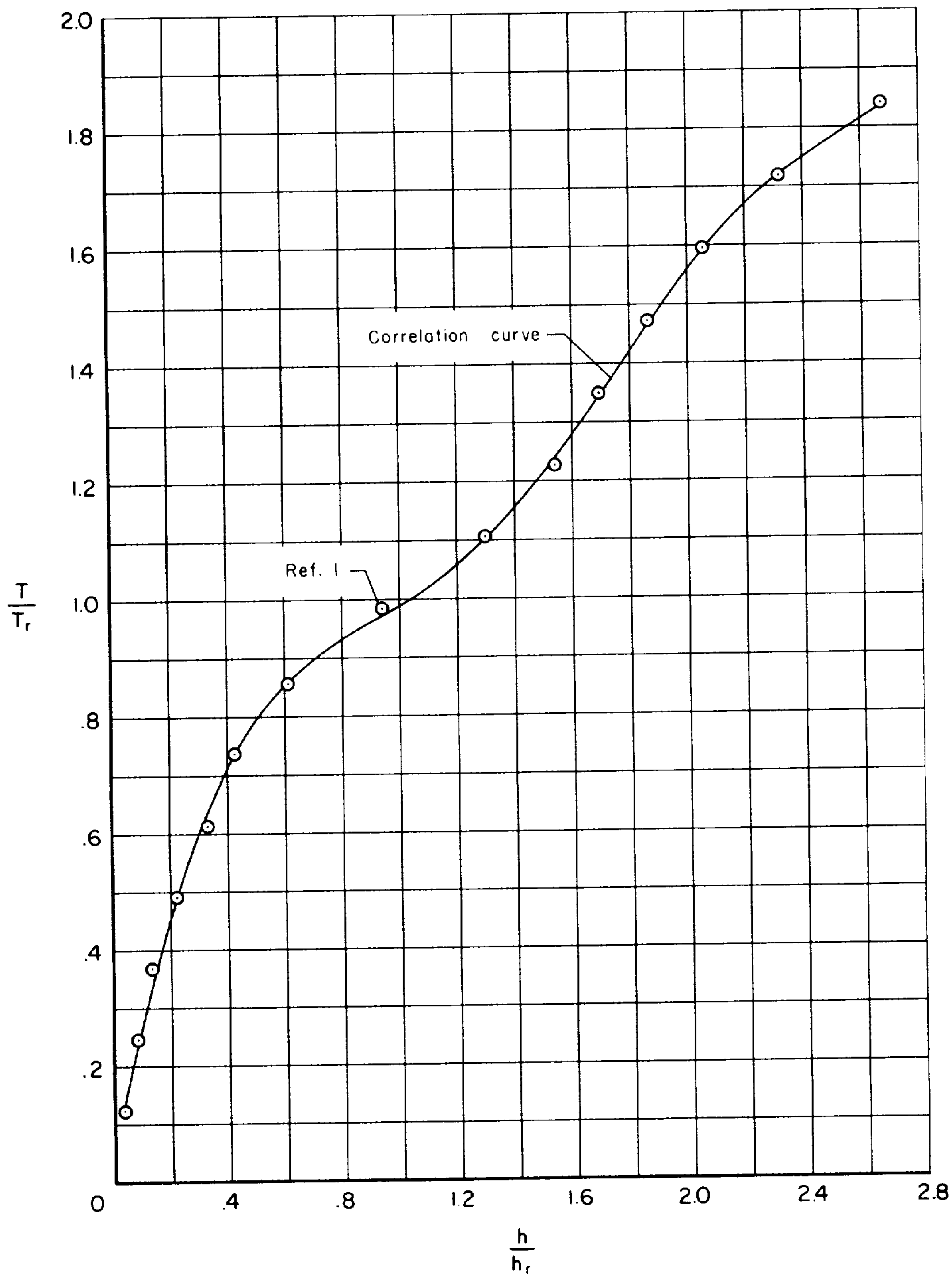
(a) $p = 10^{-1}$ atm

Figure 4.- Temperature correlation.



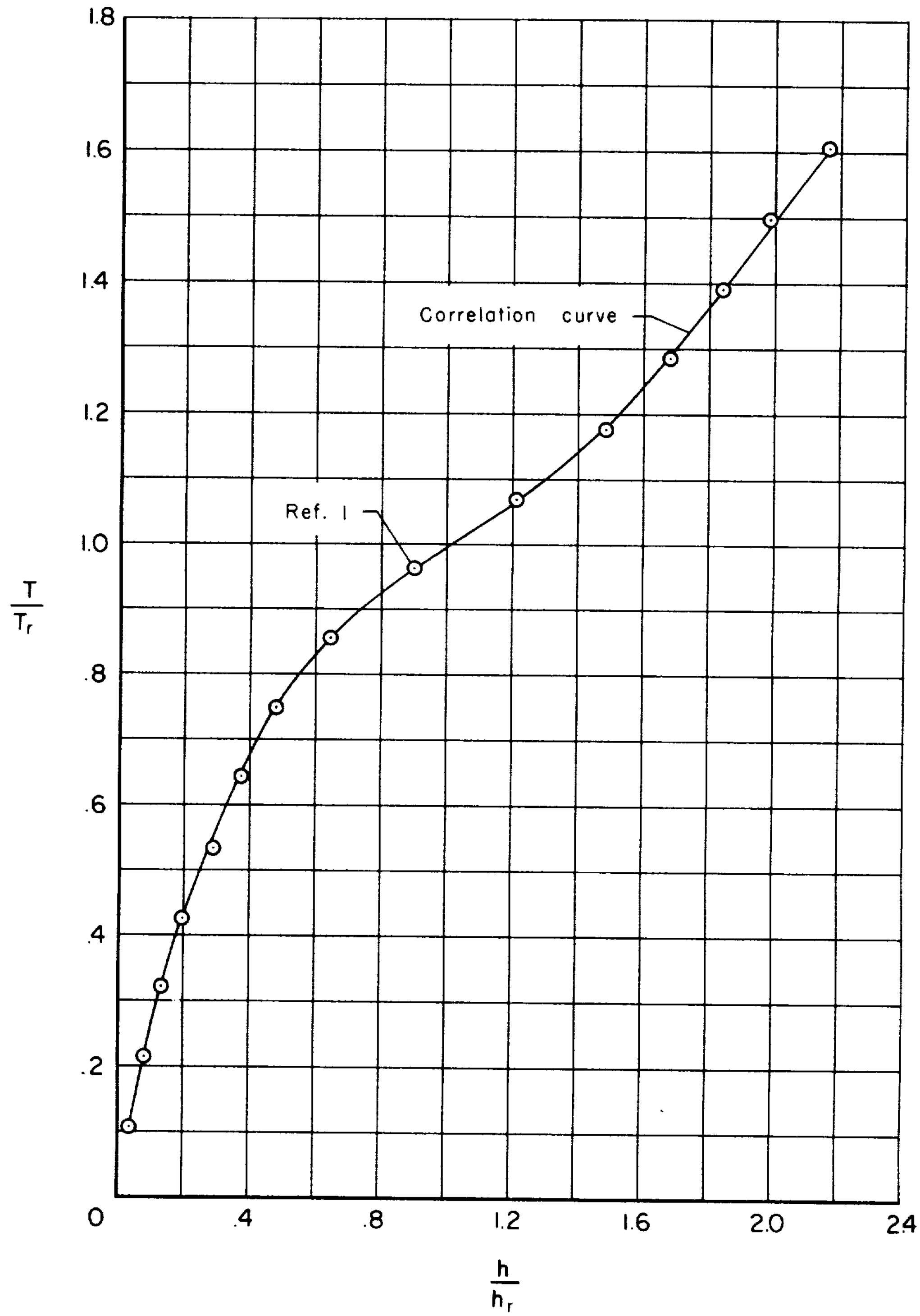
(b) p = 1 atm

Figure 4.- Continued.



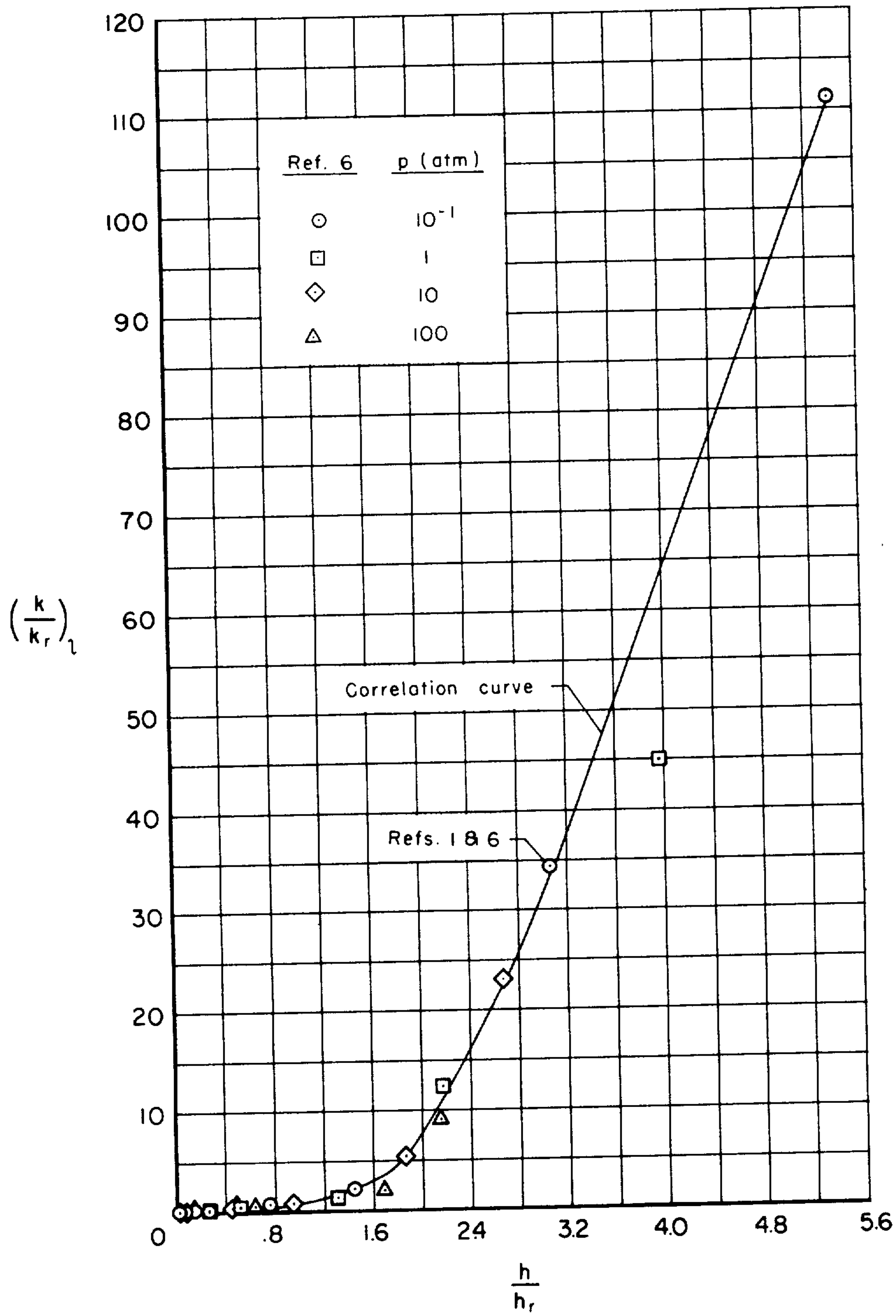
(c) p = 10 atm

Figure 4.- Continued.



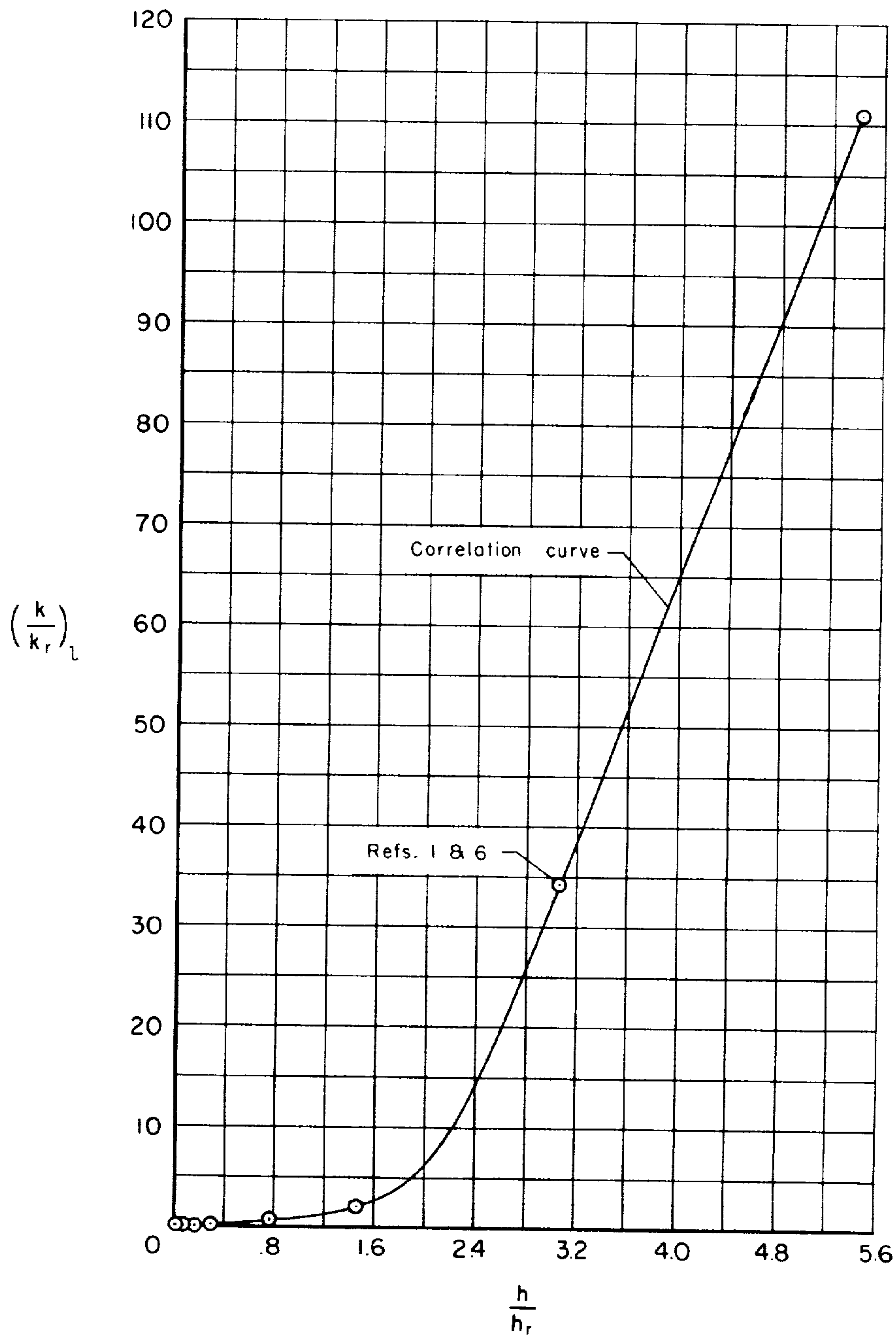
(d) p = 100 atm

Figure 4.- Concluded.



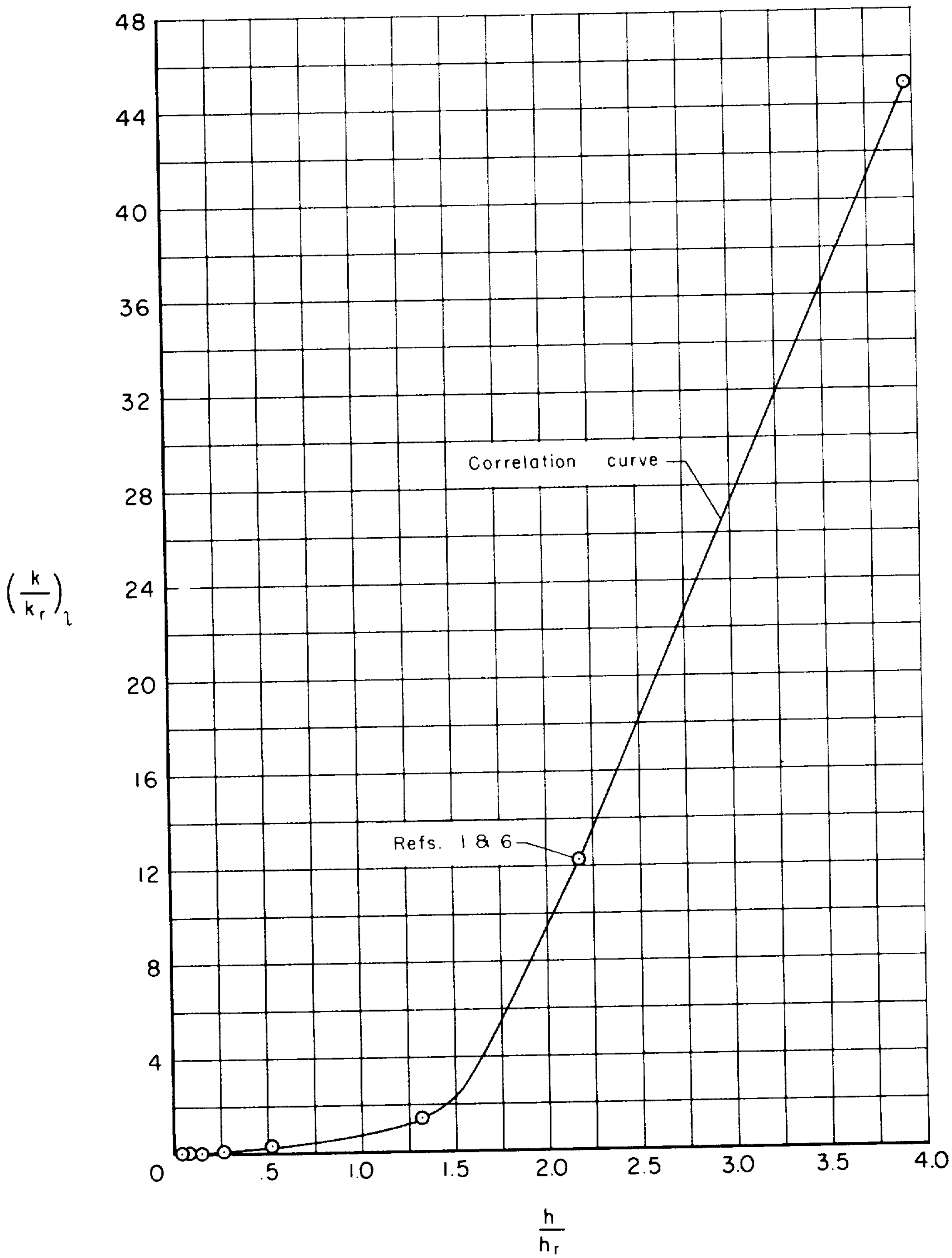
(a) All pressures.

Figure 5.- Planck mean-mass absorption coefficient correlation (ref. 6).



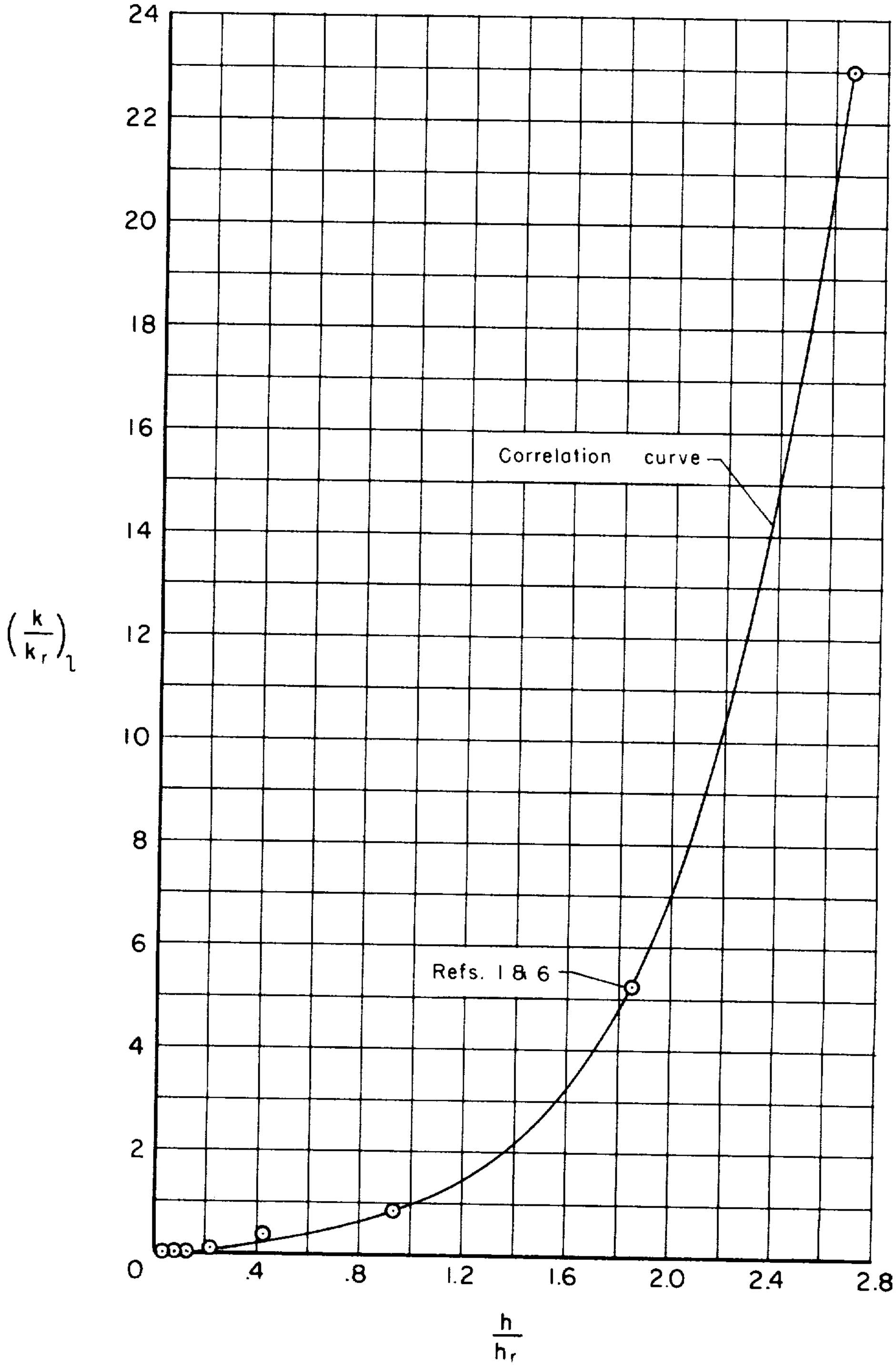
(b) $p = 10^{-1}$ atm

Figure 5.- Continued.



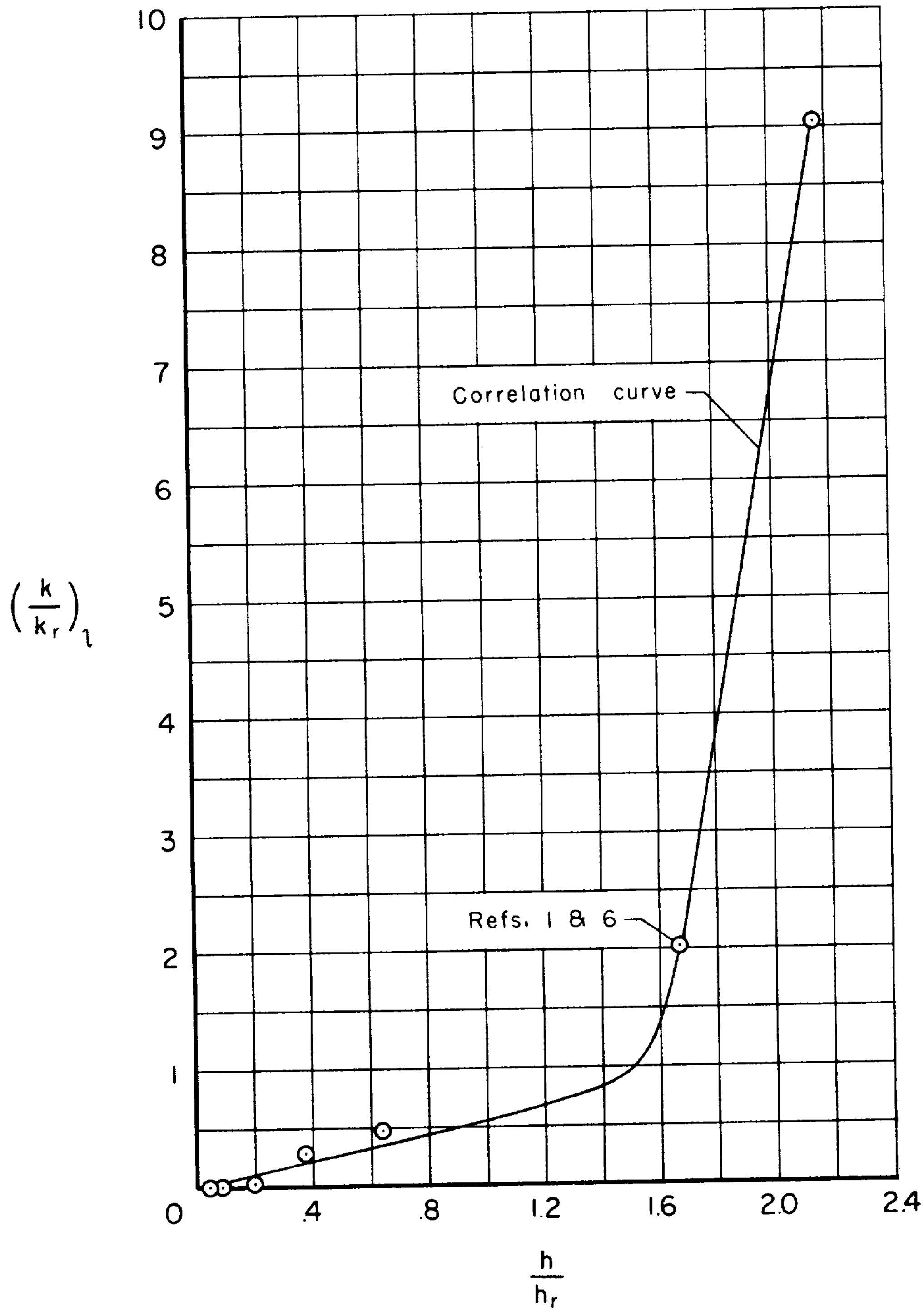
(c) p = 1 atm

Figure 5.- Continued.



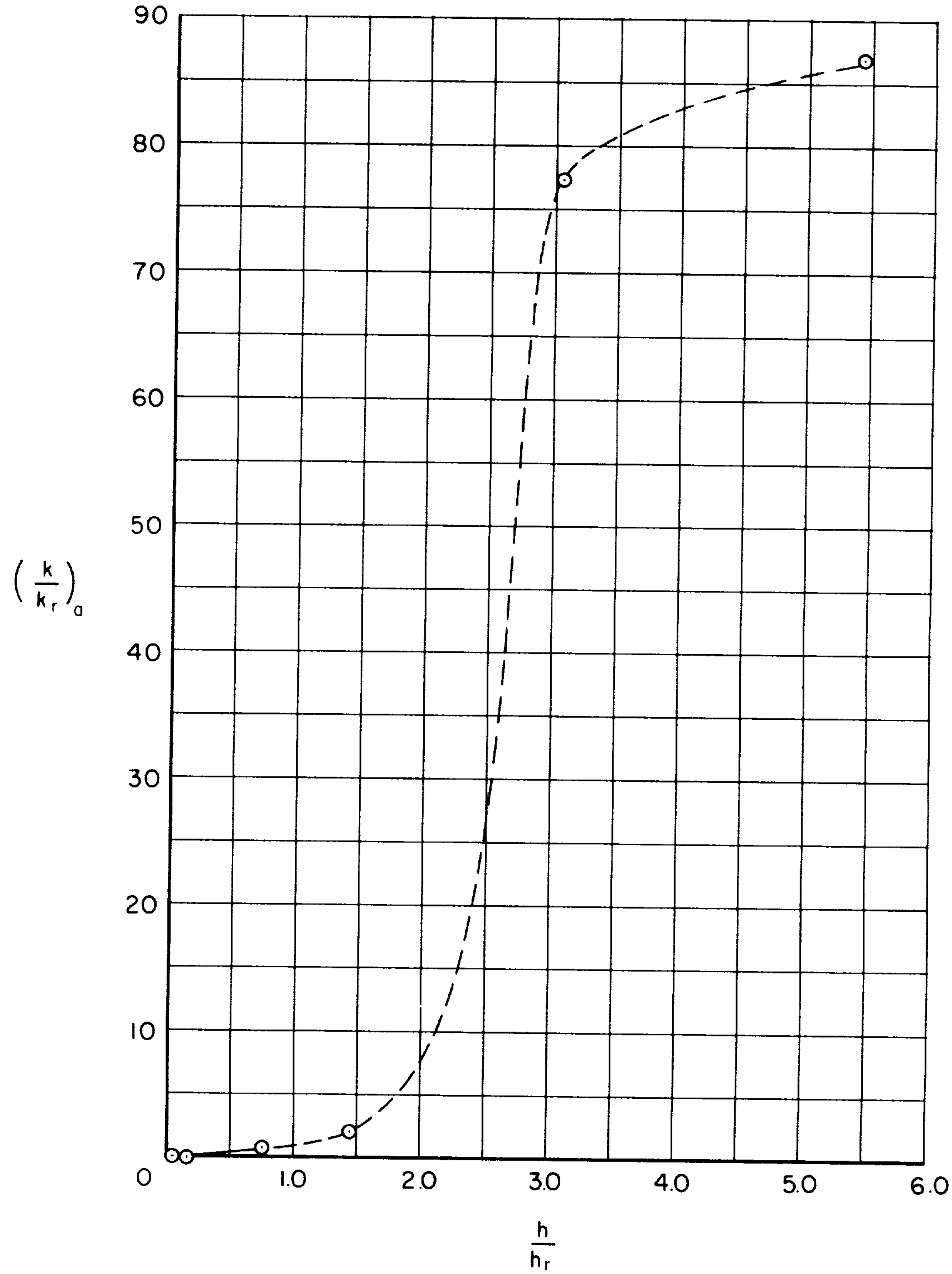
(d) p = 10 atm

Figure 5.- Continued.



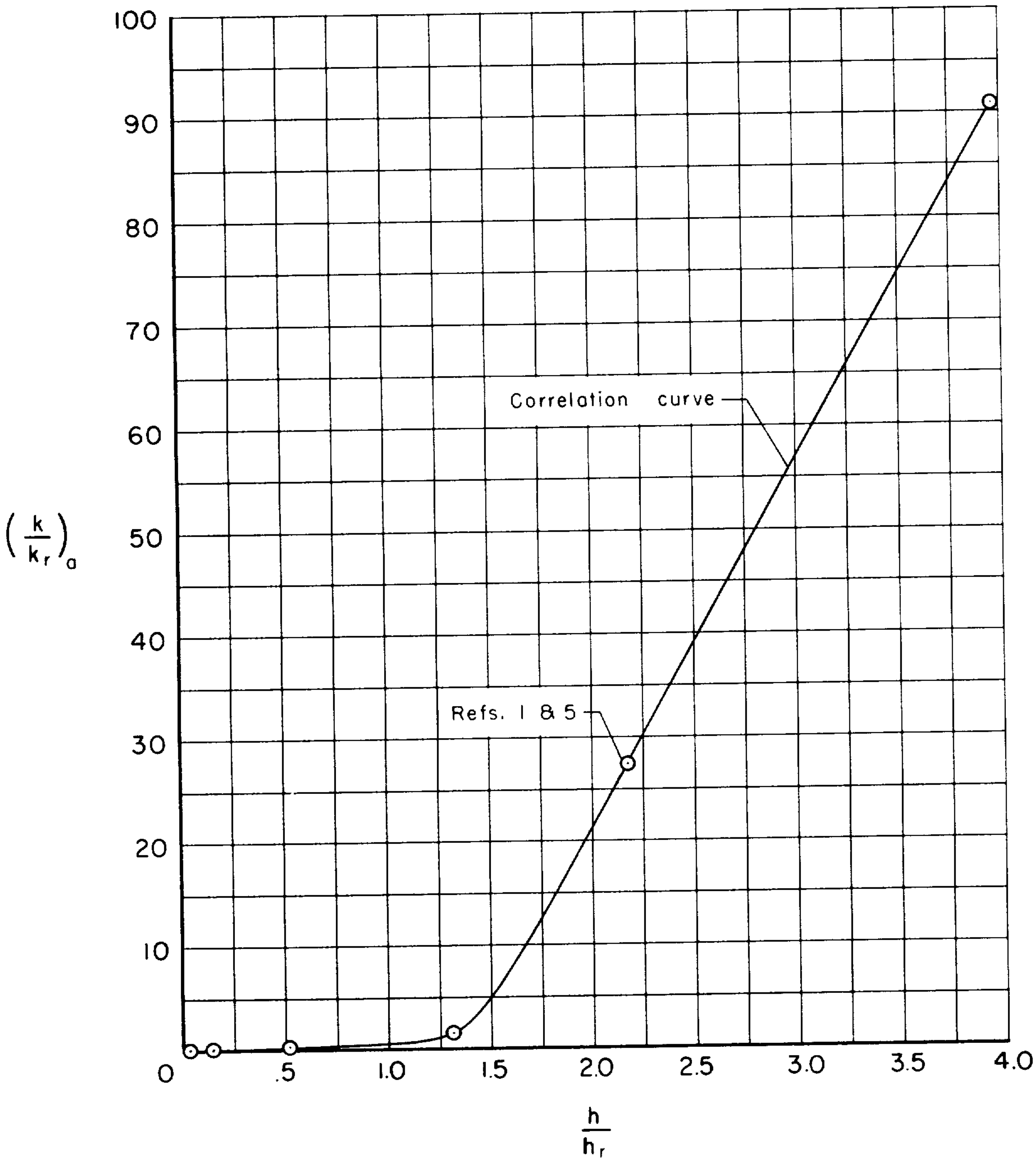
(e) p = 100 atm

Figure 5.- Continued.



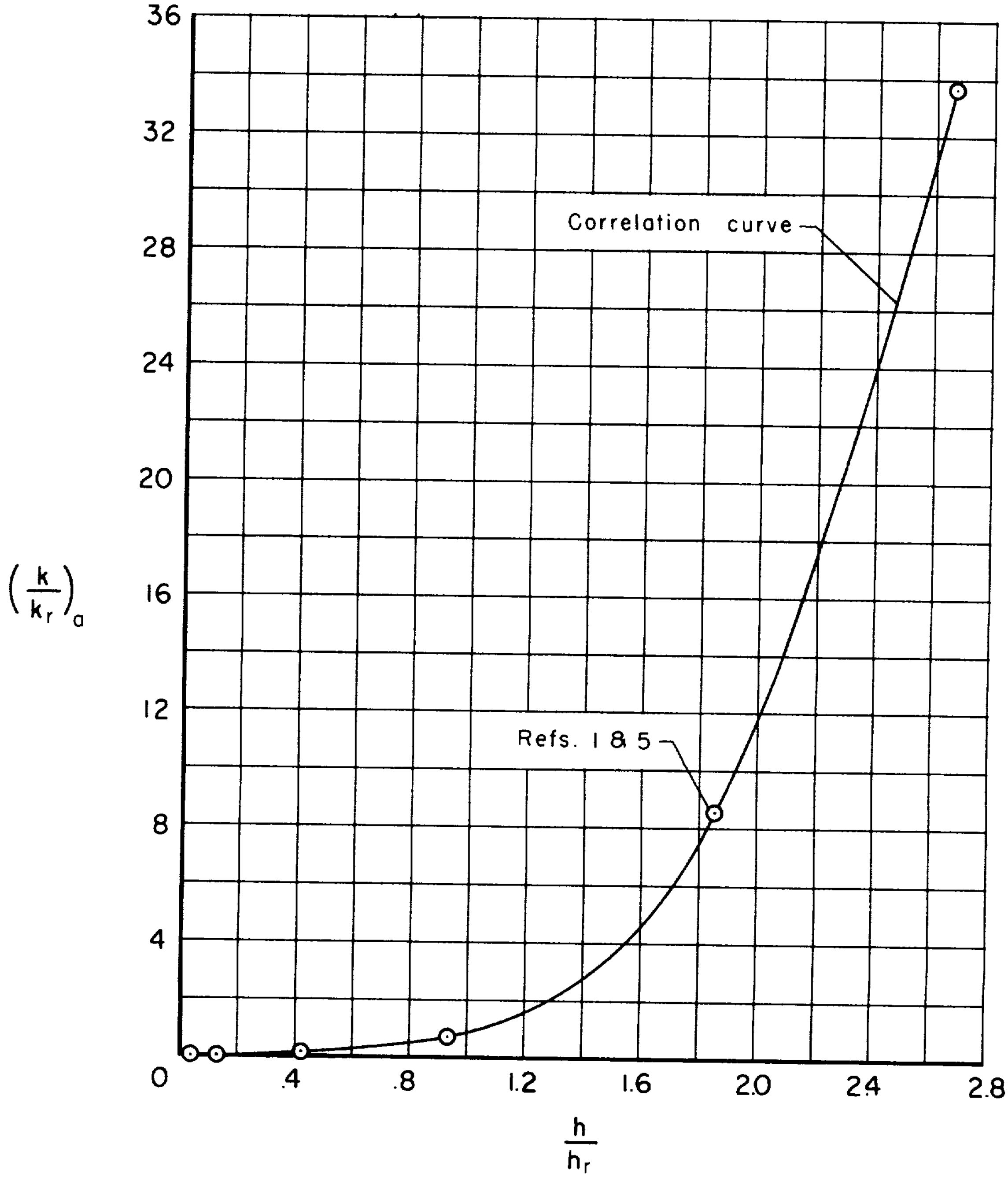
(f) $p = 10^{-1}$ atm

Figure 5.- Continued.



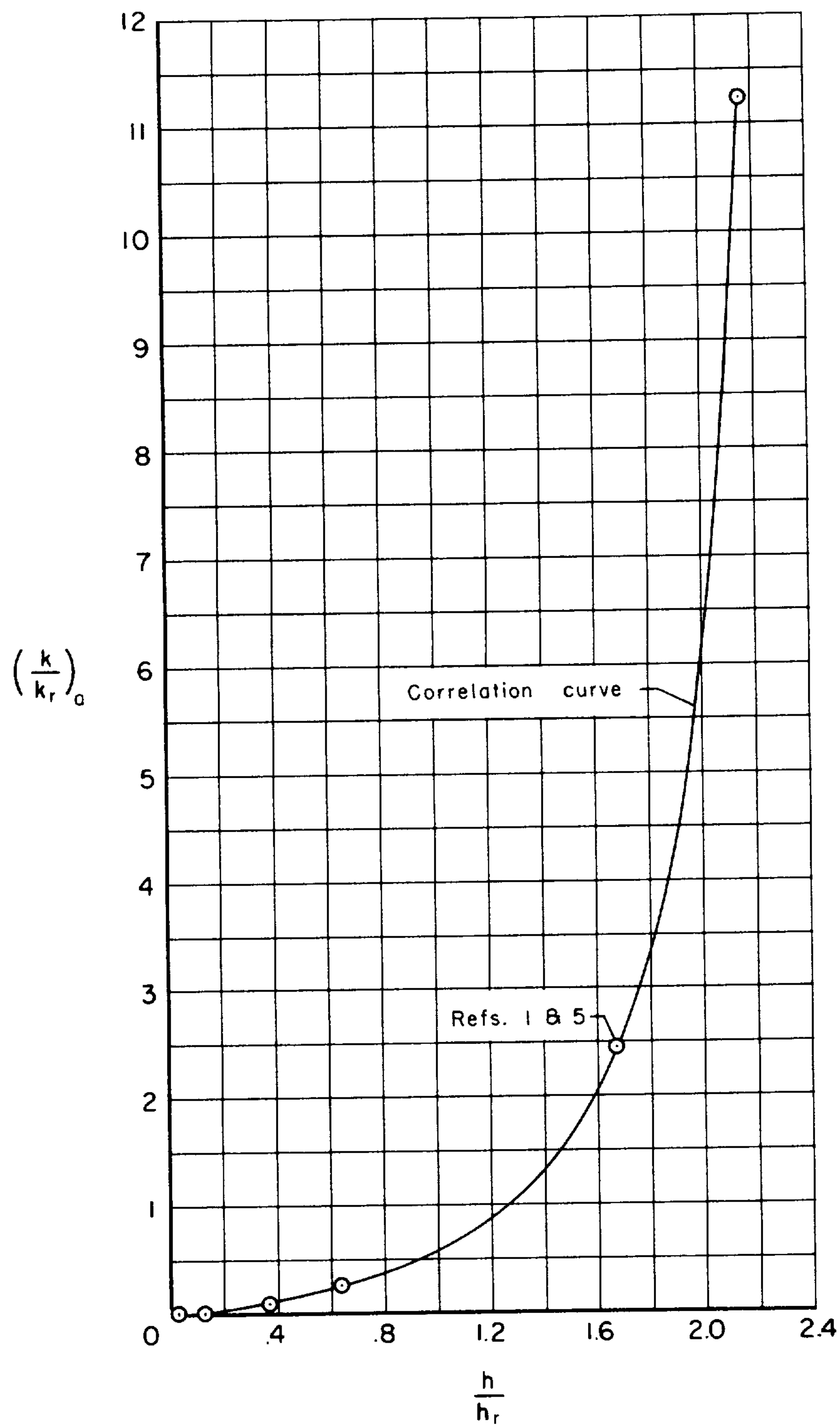
(g) p = 1 atm

Figure 5.- Continued.



(h) p = 10 atm

Figure 5.- Continued.



(i) p = 100 atm

Figure 5.- Concluded.

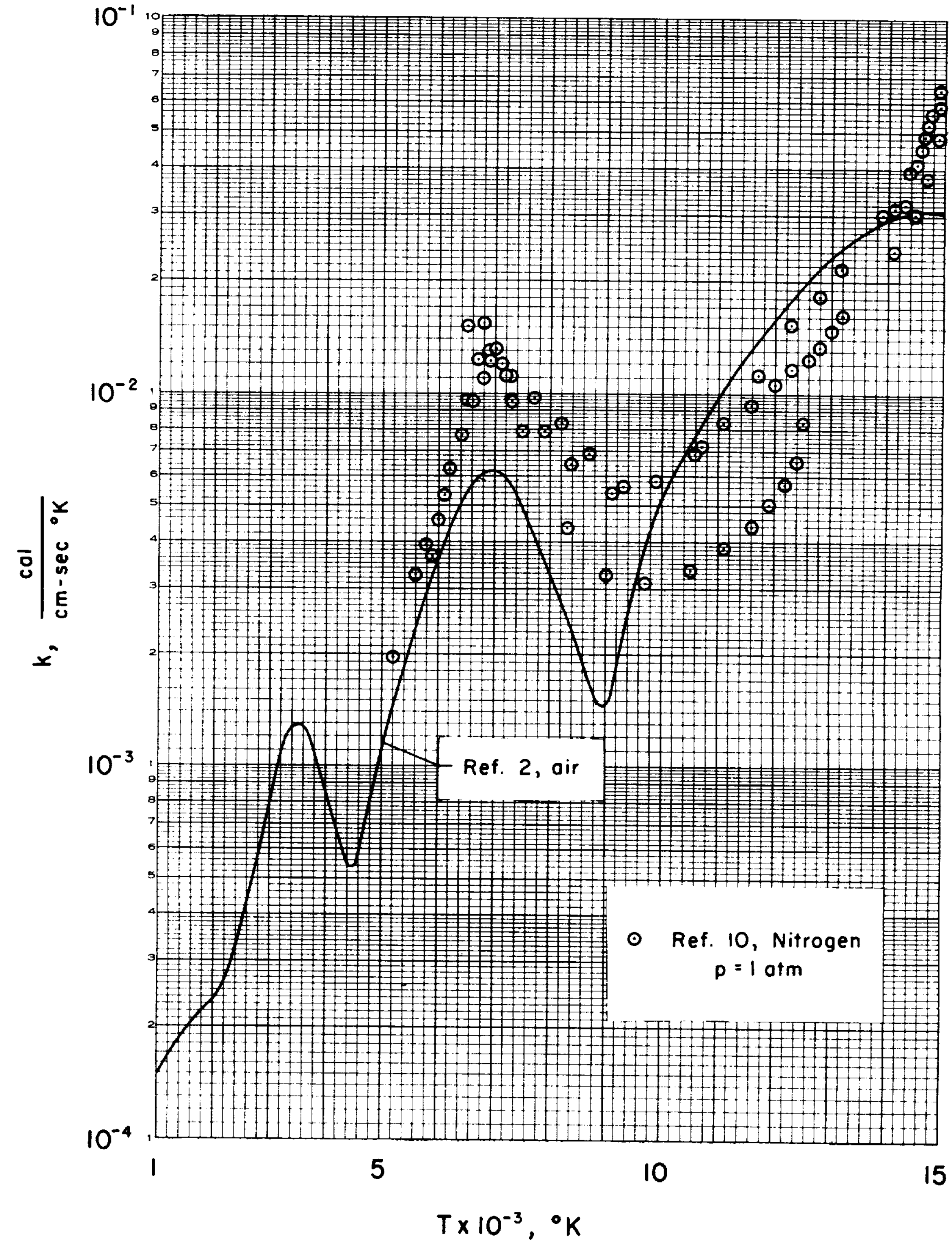


Figure 6.- Thermal conductivity - comparison of experiment and theory.